THE PATHVVAY TO KNOWLEDGE,

CONTAINING THE

First principles of *Geometrie*, as they may most aptly bee applied vnto practise, both for vse of instrumentes *Geometricall*, and *Astronomicall*: and also for proiection of plattes in euery kinde and therefore much necessarie for all sortes of men.

Geometries virdicte.

All freshe fine wittes by me are filed,
All grosse dull wittes wish me exiled:
Though no mannes witte reiect will I,
Yet as they bee, I will them trie.

IMPRINTED AT LONdon, by Iohn Harison.

for *Iohn Harison*.

1602.

THE ARGVMENTES

of the fower bookes.

The first booke declareth the definitions of the termes and names vsed in Geometrie with certaine of the chiefe groundes whereon the arte is founded. And then teacheth those conclusions, which may serue diuerselie in all workes Geometricall.

*T*he seconde booke doth sette forth the Theoremes (which may be called approued truth) seruing for the due knowledge and sure proofe of all conclusions and workes in Geometrie.

The thirde booke intreateth of diuerse formes, and sundrie protactions thereto belonging, with the vse of certaine conclusions.

The fourth booke teacheth the right order of measuring all platte formes and bodies also, by reason Geometricall.

To the Gentle Reader.

EXCVSE ME. GENtle Reader if ought bee amiſſe, ſtraunge partes are not troden all truly at the firſt: the way muſt needes be comberus, where none hath gone before. VVhere no man hath giuen light, light is it to offend, but when the light is ſhewed once, light is it to a mende. If my light may ſo light ſome other, to eſpie and marke my faultes, I wiſh it may ſo lighten them that they may voide offence. Of ſtaggering and of ſtombling and vnconſtante turmoyling: often offending and ſildome amending, ſuch vices to eſchewe, and their fine wittes to ſhew, that they may winne the praiſe, and I to hold the candle, whileſt they their glorious workes with eloquence ſet forth, ſo cunningly inuented, ſo finely indited, that my bookes may ſeeme worthy to occupie no roome. For neither is my witte ſo finely filed, neither my learning ſo largelie lettered, neither is my laiſure ſo quiet and vncumbered, that I may performe iuſtly ſo learned a laboure, or accordingly to accompliſh ſo haultie an enforcemente, yet may I thinke thus: This candle did I light: this light haue I kindeled: that learned men may ſee, to practiſe their pennes, their eloquence to aduance, to regiſter their names in the booke of memorie, I drew the platte rudely, whereon they, may build, whome God hath indued and liuelihode. For liuing by labour

doth learning so hinder, that learning serueth liuynge, which is a peruers trade yet as carefull familie shall cease hir cruell calling, and suffre anie laysure to learnynge to repaire, I will not cease from trauaile the pathe so to trade, that finer wittes may fashion them selues with such glimsing dull light, a more compleate woorke at laisure to finishe, with inuencion agreable, and aptnes of eloquence.

And this gentle Reader I hartelie protest, where erroure hath happened I wishe it redreste.

TO THE MOST
NOBLE AND PVISSAVNT PRINCE

EDVVARD THE SIXTE BY THE *grace of God, of Englande, Fraunce, & Ireland Kinge, defendour of the faith, and of the church of Englande and Irelande in earth the ſupreme head.*

IT IS NOT VNKNOVVEN to your maieſtie, moſte ſoueraigne Lorde, what greate diſceptation hath béene amongeſt the wittie men of all nacions, for the exacte knoweledge of true felicitie, bothe what it is, and wherein it conſiſteth: touchynge which thyng, their opinions almoſte were as many in numbre, as were the perſones of them, that either diſputed or wrote thereof. But and if the diuerſitie of opinions in the vulgar ſorte, for placyng of their felicitie ſhall bée conſidered alſo, the varietie ſhall be found ſo great, and the opinions ſo diſſonante, yea plainly monſterous, that no honeſt witte would vouchesafe to loſe tyme in hearyng them, or rather (as I maye ſay) no witte is of ſo exacte remembrance, that can conſider together the monſtrous multitude of theim all. And yet not withſtandyng this repugnaunt diuerſitie, in twoo thinges do they all agrée. Firſt all doe agre that felicitie is and ought to bée the ſtop and ende of all their doynges, ſo that he that hath a full deſire to any thyng, how ſo euer it be eſtemed of other men, yet he eſteemeth hym ſelf happie, if he maye obtain it: and contrary waies vnhappie if he can not attaine it. And therefore doe all men putte their whole ſtudie to gette that thyng, wherin they haue perſwaded theim ſelf that felicitie doeth conſiſte. Wherefore ſome

which put their felicitie in fedyng their beastes, thinke no pain to be harde, nor no dede to be vnhonest, that may be a meanes to fill that foule panch. Either which put their felicitie in play and idle pastymes, iudge no tyme euill spent, that is emploied thereaboute: nor no fraude vnlawfull that may further their winnyng. If I should particularly ouerrunne but the common sortes of men, which put their felicitie in their desires, it would make a greate boke of it self. Therefore will I let them all go, and conclude as I began, That all men employ their whole endeuour to that thing, wherein they thinke felicitie to stand, which thyng who so listeth to marke exactly, shall be able to espie and iudge the natures of all men, whose conuersacion he doth knowe, though they vse great dissimulacion to colour their desires, especially when they perceiue other menne to mislike that, which they so much desire: for no man would gladly haue his appetite improued. And hereof commeth that seconde thing wherein all agree, that euery man would most gladly win all other men to his secte, and to make theim of his opinion, and as farre as he dare, wil dispraise all other mennes iudgementes, and praise his owne waies onely, unles it bee when he dissimuleth, and that for the furtheraunce of his owne purpose. And this propertie also doeth giue great light to the full knowledge of mennes natures, which as al men ought to obserue, so Princes aboue other haue moste cause to marke for sundrie occasions, which may lye them on, whereof I shall not nede to speake any farther, considering not onely the greatenesse of witte, and exactnesses of iudgemente, which God hath lente vnto your highnes persone, but also the moste graue wisedom, and profound knowledge of your Maiesties moste honorable counsaile, by whom your highnes may so sufficiently vnderstande all thynges conuenient, that lesse shall it nede to vnderstande by priuate readyng, but yet not vtterly to refuse to reade as often as occasion may serue, for bokes dare speake, when menne feare to displease. But to returne againe to my firste matter, if

none

none other good thing may be learned at their manners, which so wrongfully place their felicitie, in so miserable a condition (that while they thinke them selues happy, their felicitie must needes seeme vnluckie, to be by them so euill placed) yet this may men learne at them, by those two spe ctacles to espie the secrete natures and dispositions of others which thing vnto a wise man is much auaileable. And thus will I omitte this great rablement of vnhappie hape, and will come to thrée other sortes of a better degrée whereof the on putteth felicitie to consist in power and royaltie. The second sorte vnto power anneneth worldly wisedome thinking him full happie, that could attain those two whereby he might not only haue knowledge in all thinges, but also power to bring his desire to ende. The thirde sorte estéemed true felicitie to consist in wisedome annexed with vertuous manners, thinking that they can take harme of nothing, if they can with their wisedome ouer came all vices. Of the first of those thrée sortes there hath béene a greate number in all ages, yea many mighttie kinges and greate gouernoures, which careth not great ly how they might atchieue their pourpose, so that they did preuaile. Nor did not take any greater care for gouernance then to kéepe the people in onely feare of them. Whose common sentence was alwaies this· Oderint dum metuant. And what good successe suche men had, all histories doth reporte.

But now to speake of the seconde sorte, of which there hath béene very many also, yet for this presente time amongest them all, I will take the example of kinge Philippe of Macedonie, and of Alexander his sonne that valiaunt conquerer. First that kinge Phillippe it appeareth by his letter sente vnto Aristotle that famous philosopher, that he more delited in the birth of his sonne, for the hope of learning and good education, that might happen to him him by the saied Aristotle, then he did reioyse in the continuance of his succession. for these were his words

and

and in his whole epistle, worthie to be remembred and registred euery where.

Philippe vnto Aristotle sendeth greetinge.

You shall vnderstand, that I haue a sonne borne, for which cause I yelde vnto God most heartie thankes not so much for the birth of the child, as that it was his chance to be borne in your time. For my trust is, that he shall be so brought vp and instructed by you, that he shall becom worthie not only to be named our sonne, but also to be the successour of our affaires.

And his good desire was not all vaine, for it appered that Alexander was neuer so busied with warres (yet was he neuer out of most terrible battaile (but that in the middest thereof he had in remembrance his studies, and caused in all contries as he wente, all straunge beastes,

fowles

fowles and fishes to be taken and kept by the aide of that knowledge, which he learned of Aristotle: And also he had with him alwaies a great number of learned men. And in the most busie time of all his wares against Darius King of Persia, when he harde that Aristotle had putte forthe certaine bookes of such knowledge wherein he hadde before studied, hee was offended with Aristotle, and wrote to him this letter.

Ἀλέξανδρος Ἀριστοτέλει εὖ πράττειν.

Οὐκ ὀρθῶς ἐποίησας ἐκδοὺς τοὺς ἀκροαματικοὺς τῶν λόγων, τίνι γὰρ δὴ διοίσομεν ἡμεῖς τῶν ἄλλων, εἰ καθ' οὓς ἐπαιδεύθημεν λόγους, οὗτοι πάντων ἔσονται κοινοί. ἐγὼ δὲ βουλοίμην ἂν ταῖς περὶ τὰ ἄριστα ἐμπειρίαις, ἢ ταῖς δυνάμεσι διαφέρειν. ἔρρωσο. that is,

Alexander vnto Aristotle sendeth greeting.

You haue not done well, to put forth those bookes of secrete phylosophy intituled, ἀκροαματικοί. For wherein shall we excell other, if that knowledge that we haue studied, shall be made common to all other men, namly sith our desire is to excell other men in experience & knowledge, rather then in power and strength. Farewell.

By which lettre it appeareth that hee esteemed learning and knowledge aboue power of men. And the like iudgement did he vtter, when he beheld the state of *Diogenes* Cinicus, adiudginge it the best state next to his owne, so that he said: If I were not Alexander, I would wish to bee *Diogenes*. Whereby appeareth, how he esteemed learning, and what felicity hee put therin, reputing all the worlde saue him self to bee inferiour to Diogenes And by all coniectures, Alexander did esteme *Diogenes* one of them which contemned the vaine estimation of the

deceitful world. and put his whole felicitie in knowledge of vertue, and practise of the same, though some reporte, that he knew more vertue then he followed: But what so euer he was, it appeareth that Socrates and Plato and many other did forsake their liuinges and sell a way their patrimony, to the intent to seeke & trauaile for learning which examples I shall not neede to repeate to your Maiestie, partly for that your highnes doth often reade them and other like, and partly sith your maiestie hath at hand such learned Scholemaisters, which can much bettter then I, declare thē vnto your highnes, and that more largely also then the shortnesse of this Epistle will permitte But this may I yet adde, that King Salomon whose renowme spred so farre a broade, was very greatly esteemed for his wōderful power & exceeding treasure, but yet much more was he esteemed for his wisdome. And himself doth bear witnes, that wisedome is better thē precious stones yea all things that can be desired are not to be cōpared to it. But what needeth to alledge one sētence of him, whose booke altogither do none other thing, then set forth the praise of wisdome and knowledge? And his father Kinge Dauide ioyneth vertuous cōuersation & knowledge together, as the summe of perfection, and chiefe felicitie. Wherfore I may iustly conclude, that true felicitie doth consiste in wisedome and vertue. Then if wisedome bee as Cicero defineth it, Diuinarum atque humanarum rerum scientia, then ought all men to trauale for knowledge in matters both of religion and humaine doctrine, if he shal be coūted wise and able to attain true felicitie: but as the study of religious matters is most principall, so I leaue it for this time to them that better can write of it thē I can. And for humaine knowledge, this wil I boldly say, ẏ whosoeuer will attain true iudgement therin, must not only trauail in ẏ knowledge of ẏ tongus, but must also before al other artes, tast of the Mathematicall sciences, specially Arithmetike and Geometrie, without which it is not possible to attaine

attaine full knowledge in any arte. Which may sufficiently be gathered by Aristotle not onely in his bookes of demonstration (which cannot be vnderstand without Geometrie) but also in al his other workes. And before him Plato his maister wrote this sentence on his schole house dore. nemo Geometriæ expers ingrediatur. Let no mā ēter (saith he) without knowledge in Geometrie. Wherefore most mightie prince, as your most excellent Maiestie appeareth to be borne vnto most perfecte felicitie, not onely be reason of GOD moued with the long praiers of this realme, did send your highnes as amost comfortable inheritour to the same, but also that your Maiesty was borne in the time of so skilfull schole maisters and learned teachers, as your highnes doth not a little reioyce in, & profite by them in al kind of vertue and knowledge. Amongst which is that heauenly knowledge most worthilie to bée praised, whereby the blindnes of errour and superstition is exiled, and good hope receiued ẏ all the séedes & fruites thereof, with all kinde of vice and iniquite, whereby vertue is hindered, & iustice defaced, shall be cleane extirped & rooted out of this realme, which hope shal increase more and more, if it may appear that learning be estéemed and florish within this realm. And all be it the chief learning be the diuin Scriptures, which instruct the mind principally, and next therto the lawes politik, which most specially defend the right of goodes, yet is it not possible, that those two can long be well vsed, if that aid want that gouerneth health and expelleth sicknes, which thing is don by Phisicke, and these requir the help of the seuen liberall sciences, but of none more then of Arithmetike and Geometrie, by which not onely greate thinges are wroughte touching accomptes in al kindes, and in suruaying & measuring of landes but also all artes depend partly of them, and building which is most necessary can not be without them which thing cōsidering moued me to helpe to serue your maiestie in this pointe, as wel as other waies, an to

do what may be in me, that not ōly they which study prin cipally for learning, may haue furderāce by my poore help but also those which haue no time to trauaile for exacter knowledge may haue some helpe to vnderstand these Mathematicall artes, in which as I haue all readie sette forth somwhat of Arethmetike, so God willing I intend shortly to set forth a more exacter worke thereof. And in the meane season for a tast of Geometrie, I haue sette forth this smal introduction, desiring your grace not so much to behold the simplenes of the worke in comparison to your Maiesties excellence, as to fauour the edition therof for the aide of your humble subiectes, which shall thinke themselues more & more dayly boūde to your highnes, if when they shall perceiue your graces desire to haue them profited in all knowledge and vertue. And I for my poore ability cōsidering your maiesties study for ẏ increse of learning generally through al your highnes dominions, and namly in the vniuersities of Oxforde and Cambrige, as I haue an earnest good will as far as my simple seruice and small knowledge will suffice, to helpe toward the satisfīng of your graces desire, so if I shall perceaue that my seruice may be to your maiesties contentacion, I wil not only put forth the other two bookes, which should haue beene sette forth with these two, if misfortūe had not hindered it, but also I will sette forth other bookes of more exacter arte, both in the Latine tonge and also in the English, whereof parte bee alreadie written, and new instrumentes to them deuised, and the residue shall bee ended withall possible speede. I was boldened to dedicate this booke of Geometrie vnto your Maiestie, not so much because it is the first that euer was sette forth in English, and therefore for the noueltie a straunge present, but that I was perswaded, that such a wise prince doth desire to haue a wise sorte of subiectes. For it is a kinges chiefe reioysing and glorie, if his subiectes be rich in substance, and wittie in knowledge

and

and contrarie wise nothing can bee more greuouse to a noble King, then ẏ his Realme should be ether beggerlie or full of ignorannce: But as God hath giuen your grace a realme both riche of commodities and also full of witty men, so I trust by the reading of wittie artes (which be as the whette stones of witte) they must nedes increase more and more in wisdome, and parad-efture finde some thing towarde the aide of their substance, where by your grace shall haue newe occation to reioyce, seing your subiectes to increase in substance or wisdome or in both. And they againe shall haue new & new causes to pray for your Maiestie, perceiuing so gracious a mine towarde their benifite. And I trust (as I desire) that a greate number of gentlemen, specially a boute the court, which vnderstād not the Latine tong, or els for the hardnesse of the matter could not a way with other mens writing, will fall in trad with this easie forme of teaching in their vulgar tōg and so imploy some of their time in honest studie, which were wonte to bestow most parte of their time in trifling pastime for vndoubtedly if they meane either your maiesties seruice, ether their own wisdome they wil be cōtent to imploy some time about this honest and wittie exercise. For whose encouragement to the intent they may perceiue what shall be the vse of this science, I haue not onely written somewhat of the vse of Geometrie but also I haue annexed to this boke the names and brefe argumentes of those other bokes which I haue sette forth hearafter, and that as shortly as it shal appeare vnto your Maiestie by coniecture of their diligent vsing of this first boke, that they will vse well the other bokes also In the meane season, and at all times I will be a continuall petitioner, that God may worke in al English hartes, an ernest mine to all honest exercises, whereby they may serue the better your Maiestie and the Realme. And for your highnes I beseech the most merciful God, as he hath most fauourably sent you vnto vs, as our chiefe comforter in

earth, so that he will increase your Maiestie dayly in all vertue and honer with most prosperous successe, and augment in us your most humble subiectes, true loue to god ward, and iust obedience toward your highnes with all reuerence and subiection.

At London the xxvlij. day of January. M.D.L.I.

Your Maiesties most humble seruant and obedient subiect, Robert Recorde.

THE PREFACE.

declaring briefly the commodities of Geometrie, and the necessitie thereof

GEOMETRIE maie thinke it self to sustaine greate iniurie, if it shal bee inforced either to shewe her manifolde commodities, or els not to prease into the sight of menne, and therefore mighte this waies answere briefly: Either I am able to doe you muche good, or els but little. If I bee able to doe you muche good, then bee you not your owne friendes, but greatly your owne enemies, to make so little of me, whiche maie profite you so muche. For if I were as vncurteous, as you vnkinde, I should vtterly refuse to do them any good, whiche will so curiously put me to the triall and profe of my commodities, or els to suffer exile, and namely sith I shall onely yelde benefites to other, and receiue none againe. But and if you could saie truely, that my benefites bee neither many, nor yet greate, yet if they bee any, I doe yelde more to you, then I doe receiue againe of you, and therefore I ought not to be repelled of them that loue them selfes, although they loue me not at al for my self. But as I am in nature a liberall science, so can I not againste nature contende with your inhumanitie, but muste shewe my self liberall euen to myne enemies. Yet this is my comfort againe, that I haue none enemies, but them that know me not, and therefore maie hurte them selues, but can not annoye me. If they dispraise the thyng that they knowe not, all wise men will blame them, and not credite them. And if they thinke they knowe me, let them shewe one vntruthe and errour in me, and I will giue the victorie.

Yet can no humayne Science saye thus, but I onely, that there is no sparke of vntruthe in me: but all my doctrine and workes are without any blemishe of errour, that mannes reason can discerne. And nexte vnto me in certaintie are my thre sisters, Arithmetike, Musike, and Astronomie, whiche are also so nere knitte in amitie, that he that loueth the one, can not despise the other, and in especiall, Geometrie, of whiche not onely these thre, but all other artes doe borowe greate aide, as partly hereafter shall be shewed. But first I will beginne with the vnlearned sorte, that you maye perceiue how that no arte cā stand without me. For if I should declare how many wayes my helpe is vsed, in measuryng of ground, for medowe, corne, and wood: in hedging in dichyng and in stakes makyng, I thinke the pore Husbande manne would bee more thankefull vnto me, then he is now, whiles he thinketh that he hath small benefite by me. Yet this may he conjecture certainly, that if he kepe not the rules of Geometrie, he can not measure any grounde truely. And his dichyng, if he keepe not a proportion of bredth in the mouthe, to the bredh of the bottome, & iuste sloapenesse in the sides, agreable to them bothe, the diche shalbe faultie many waies. When he doeth make stackes for corne, or for heye, he practiseth good Geometrie, els would they not long stand: so that in some stackes, which stande on fower pillers, and yet made rounde, doe increase greater and greater a good heighte, and then again turne smaller and smaller vnto the top: you may see so good Geometrie, that it were verye difficulte to counterfaite the like in any kynde of buildyng. As for other infinite waies that he vseth my benefite, I ouerpasse for shortnesse.

Carpenters, Karuers, Joyners, and Masons, doe willingly acknowledge, that they can worke nothyng without reason of Geometrie, in so muche that they chalenge me as a peculiare science for thē. But in that they should doe wrong to all other men, seyng euery kinde of men haue some benefite by me, not onely in buildyng, whiche is but other mens costes, and the arte of Carpenters, Masons, and other aforesaied,

saied, but in their owne priuate profession, whereof to auoide tediousnesse I make this rehearsall.

Sith Merchauntes by Shippes greate riches doe winne,
I maie with good right at theire feete beginne.
The Shippes on the sea with Saile and with Ore,
Were first founde, and still made, by *Geometries* lore.
Their Compas, theire Carde, their Pulleis, their Ankers,
were founde by the skill of wittie *Geometers*.
To sette forthe the Capstocke, and eche other parte,
would make a greate showe of *Geometries* arte.
Carpenters, Karuers, Ioyners and Masons,
Painters and Limmers with suche occupations,
Broderers, Goldsmithes, if they bee cunnyng,
Must yeelde to *Geometrie* thankes for their learnyng.
The Carte and the Plowe, who doeth them well marke,
Are made by good *Geometrie*. And so in the warke
Of Tailers and Shoomakers, in all shapes and fashion,
The woorke is not praised, if it wante proportion.
So weauers by *Geometrie* had their foundation,
Theire Loome is a frame of straunge imagination.
The wheele that doeth spinne, the stone that doeth grinde,
The Mill that is driuen by water or winde,
Are woorkes of *Geometrie* straunge in their trade,
Fewe could them deuise, if they were vnmade.
And all that is wrought by waight or by measure,
without proofe of *Geometrie* can neuer be sure.
Clockes that be made the tymes to deuide,
The wittiest inuention that euer was spied,
Now that they are common they are not regarded,
The artes man contemned, the woorke vnrewarded.
But if they were scarse and one for a showe,
Made by *Geometrie*, then should men knowe,
That neuer was arte so wonderfull wittie,
So needefull to man, as is good *Geometrie*.
The first findyng out of euery good arte,
Seemed then vnto men so godlie a parte,

That no recompence might satisfie the finder,
. But to make hym a God, and honour hym for euer,
So *Ceres* and *Pallas*, and *Mercurie* also,
Eolus and *Neptune*, and many other mo,
Were honoured as godds, because they did teache,
First tillage and weauyng, and eloquent speach,
Or windes to obserue, the seas to saile ouer,
They were called godds for their good indeuour.
Then were men more thankfull in that golden age:
This yron worlde now vngratefull in rage,
Will yeelde thee thy reward for trauaile and paine,
With slaunderous reproche, and spitefull disdaine.
Yet though other men vnthankfull will be,
Suruayers haue cause to make muche of me.
And so haue all Lordes that landes doe possesse:
But Tenauntes I feare will like me the lesse.
Yet doe I no wrong but measure all truely,
And yeelde the full right to euery man iustely.
Proportion *Geometricall* hath no man opprest,
If any bee wronged, I wishe it redrest.

But now to procede with learned profession, in Logike and Rhetorike, and all partes of Philosophie, there needeth none other proofe then Aristotle his testimonie, whiche without Geometrie proueth almoste nothyng. In Logike all his good syllogismes and demonstrations, be declared by the principles of Geometrie. In Philosophie, neither motion, nor tyme, nor ayrie impressions, could he aptly declare, but by the helpe of Geometrie, as his workes doe witnesse. Yea the faculties of the mynde doeth he expresse by similitude, to figures of Geometrie. And in moral Philosophie he thought that Justice could not be well taught, nor yet well executed without proportion Geometricall. And this estimation of Geometrie he maie seme to haue learned of his maister Plato, which without Geometrie would teache nothyng, neither admitte any to heare hym, excepte he were experte in Geometrie. And what merueill if he so muche estemed Geo-

metrie,

metrie, seyng his opinion was, that God was alwaies workyng by Geometrie: Which sentence Plutarche declareth at large. And although Plato doe vse the helpe of Geometrie in all the moste waightie matters of a common wealthe, yet it is so generall in vse, that no small thynges can bée well doen without it. And therefore saieth he: that Geometrie is to be learned, if it weare for none other cause, but that al other artes are bothe sooner & more surely vnderstood by help of it.

What greate helpe it doeth in Phisike, Galen doeth so often and so copiously declare, that no man which hath red any booke almoste of his, can bée ignoraunte thereof. In so much that he could neuer cure well a round vlcer, till reason Geometricall did teach it him. Hippocrates is earnest in admonishyng that studie of Geometrie, must prepare the waie to Phisike, as well as to all other artes.

I should séeme somwhat to tedious, if I should recken vp, how the diuines also in their misteries of scripture, doe vse helpe of Geometrie: and also that lawyers can neuer vnderstande the whole lawe, no nor yet the firste title thereof exactly without Geometrie. For if Lawes can not well bée established, nor iustice duely executed without Geometrical proportion, as both Plato in his Politike bookes, and Aristotle in his Moralles doe largely declare. Yea sith Lycurgus that chiefe lawmaker amongest the Lacedemonians, is most praised, for that he did chaunge the state of their Common wealth, from the proportion Arithmeticall, to a proportion Geometricall, which without knowledge of both he could not do, then is it easie to perceiue how necessarie Geometrie is for the lawe, and studentes thereof. And if I shall saie precisely and fréely as I thinke, he is vtterly destitute of all abilitie to iudge any arte, that is not some what experte in the Theoremes of Geometrie. And that caused Galene to saie of hym self, that he could neuer perceiue what a demonstratiō was, no not so much, as whether there were any or none, till he had by Geometrie gotten abilitie to vnderstande it, al though he heard the beste teachers that were in his tyme.

It should be to long and nedelesse also to declare, what helpe all other artes Mathematicall haue by Geometrie, sith it is the ground of all their certaintie, and no man studious in thē is so doubtfull thereof, that he shall néede any persuasion to procure credite thereto. For he can not read ij. lines almoste in any Mathematicall science, but he shall espie the nedefulnes of Geometrie. But to auoide tediousnes I wil make an ende hereof with that famous sentēce of auncient Pythagoras, *That who so will trauaile by learnyng to attaine wisedome, shall neuer approche to any excellencie without the artes* Mathematicall *and especially* Arithmetike *and* Geometrie.

And if I shall some what speake of noble men, and gouernours of realmes, how néedefull Geometrie maye bée vnto them, then must I repete all that I haue sayed before, sith in them ought all knowledge to abound, namely that maye appertayne either to good gouernaunce in tyme of peace, either wittie pollicies in tyme of warre. For ministration of good lawes in tyme of peace Lycurgus example, with the testimonies of Plato & Aristotle maye suffice. And as for warres, I might thinke it sufficiente that Vegetius hath written, and after hym Valturius in commendation of Geometrie, for vse of warres, but all their wordes seme to saye nothyng, in comparison to the example of Archimedes worthy workes made by Geometrie, for the defence of his Countrey, to reade the wonderfull praise of his wittie deuises, set forthe by the moste famous histories of Liuius, Plutarche, and Plinie, and all other historiographers, whiche write of the strong siege of Syracusa, made by that valiaunte Captaine, and noble warriour Marcellus, whose power was so greate, that all menne meruailed how that one Citie could withstande his wonderfull force so longe. But muche more would they meruaile, if they vnderstode that one man onely did withstande all Marcellus strength, and with counter engines destroied his engines, to the vtter astonishement of Marcellus, and all that were with hym. He had inuented suche balastelas that did shote out a hundred dartes at one

shote

shote, to the greate destruction of Marcellus Souldiours, whereby a fonde tale was spred abrode, how that in Syracusa there was a wonderfull Gyante, whiche had an hundre handes, and could shote a hūdred dartes at once. And as this fable was spred of Archimedes, so many other haue been faned to be gyantes and monsters, because they did suche thinges, whiche farre passed the witte of the common people. So did they feigne Argus to haue an hundred eyes, because they heard of his wonderfull circumspection, and thought that as it was aboue their capacitie, so it could not be, vnlesse he had a hundred eyes. So imagined they Ianus to haue twoo faces, one loking forward, and an other backward, because he could so wittily compare thynges past, with thynges that were to come, and so duely ponder them, as if thei were all presente. Of like reasō did thei feyn Lynceus to haue suche sharp sight that he could se through walles and hilles, because peraduēture he dyd by naturall iudgement, declare what cōmodities myght be digged out of the grounde. And an infinite nomber like fables are there, whiche sprange all of like reason.

For what other thyng meaneth the fable of the greate gyante Atlas, whiche was imagined to beare vp heauen on hys shoulders: but that he was a man of so hygh a witte, that it reached vnto the skye, and was so skilfull in Astronomie, and could tell before hande of Eclipses, and other like thynges, as truely as though he did rule the starres, and gouerne the Planettes.

So was Eolus accompted God of the windes, and to haue thē all in a caue at his pleasure, by reason that he was a wittie man in naturall knowledge, & obserued well the chaunge of weathers, and was the first that taught the obseruatiō of the windes. And like reason is to be giuē of all the old fables.

But to retourne againe to Archimedes, he did also by art perspectiue (whiche is a part of Geometrie) deuise suche glasses within the towne of Syracusa, that did burne their ennemies shippes a greate waie from the towne, which was a meruailous politike thynge. And if I should repeate the

varietie,

varieties of such straunge inuentions, as Archimedes and others haue wrought by Geometrie, I should not onely excede the order of a Preface, but I should also speake of suche thinges as can not well bee vnderstoode in talke, without some knowledge in the principles of Geometrie.

But this will I promise, that if I maie perceiue my paines to bee thankfully taken, I will not onely write of suche pleasaunte inuentions, declaryng what they were, but also will teach how a greate nomber of them were wroughte, that they maie be practised in this tyme also. Whereby shalbe plainly perceiued, that many thynges seeme impossible to bee done, which by arte maie verie well bee wrought. And when they bee wrought, and the reason thereof not vnderstoode, then saie the vulgare people, that those thynges are doun by Negromancie. And hereof came it that Frier Bakon was accompted so greate a Negromancier, whiche neuer vsed that art (by any coniecture that I can finde) but was in Geometrie, and other Mathematicall sciences so experte, that he could doe by them suche thynges as were wonderfull in the sight of moste people.

Great talke there is of a glasse that he made in Oxforde, in whiche mē might see thinges that weare doen in other places, and that was iudged to bee doen by power of euill spirites. But I knowe the reason of it to bee good and naturall, and to be wrought by Geometrie (sith perspectiue is a parte of it) and to stande as well with reason, as to see your face in common glasse. But this conclusion and other diuers of like sort, are more meete for Princes, for sundrie causes, then for other men, and ought not to be taught commonly. Yet to repeate it, I thought good for this cause, that the worthines of Geometrie might the better be knowen, & partly vnderstāding giuen, what wonderfull thinges maie be wrought by it and so consequently how pleasant it is, & how necessary also.

And thus for this tyme I make an ende. The reason of some thynges doun in this booke, or omitted in the same, you shall finde in the Preface before the Theoremes.

THE DEFINITIONS of the principles of GEOMETRIE.

GEOMETRIE teacheth the drawyng, Measuring, and proportion of figures, but in as muche as no figure can bee drawen, but it muste haue certaine boundes & inclosure of lines: and euery line also is begun and ended at some certaine pricke, firste it shall bee meete to knowe these smaller partes of euery figure, that therby the whole figures maie the better be iudged, & distincte in sōder. A pointe,

A Point or a Pricke, is named of the Geometricians that small and unsensible shape, whiche hath in it no partes, that is to saie: neither length, breadth, nor depth. But as this exactnes of definition, is more meeter for onely Theorike speculation, then for practise, and outwarde woorke (considering) that myne intente is to applie all these whole principles to woorke) I thinke méeter for this purpose, to call a point or pricke, that small print of penne, pencile, or other instrumēt, which is not moued, nor drawne from his firste touche, and therefore hath no notable length nor breadth: as this example doeth declare. ∴

Where I haue set .iij. prickes, each of them hauyng bothe length, and breadth, though it be but small, and therefore not notable.

Now of a greate number of these prickes, is made a line, as you may perceiue by this forme ensuyng. ....
Where as I haue set a nōber of prickes, so if you with your penne, will set in more other prikes betweene euery two of these, then will it be a line, as here you may see ------------
and this line, is called of Geometricians, length without bredth. A line.

But as they in their Theorikes (which are onely in mynde

workes) doe precisely vnderstande these definitions, so it shalbe sufficiente for those men which seeke the vse of the sāe thinges, as sense may duelie iudge them, and applie to handie workes, if they vnderstand them so to be true, that outwarde sense can finde none errour therein.

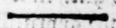
Of lines there be two principall kindes. The on is called a right, or straight line, and the other a crooked line.

A streight line.

A straight line, is the shortest that may be drawne betwene two pricks.

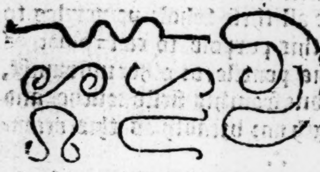
A crooked line

And all other lines, that goe not right forth from pricke to pricke but boweth any way, such are called crooked lines as in these examples following, ye may see, where I haue set but one forme of a straight line, for more formes there be not, but of crooked lines there bee innumerable diuersities, whereof for examples, some I haue sette here.



A right line

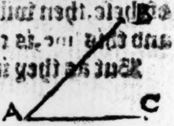
Crooked lines.



Crooked lines.

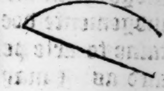
So now you must vnderstande, that euerie line is drawē betwene two prickes whereof the one is at the beginning, and the other at the ende.

Therefore, when so euer you doe see any formes of lines, to touche at one notable pricke, as in this example, then shall



you

you not call it one crooked line, but rather two lines; in as much as there is a notable and ſenſible angle by A. which euer more is made the meeting of two ſe‑uerall lines. And likewiſe ſhall you iudge of this figure which is made of two lines, & not of one onely.



An Angle

So that when ſo euer any ſuch meeting of lines doeth happen, the place of there meeting is called an angle or corner

Of angles there bée thrée generall kindes: a ſharpe angle, a ſquare angle and a blunte angle The ſquare angle which is commonly named a right corner, is made of two lines meeting together in forme of a ſquire, which two lines if they be drawen forth in length, will croſſe one an other: as in the examples following you may ſée.

A right angle.

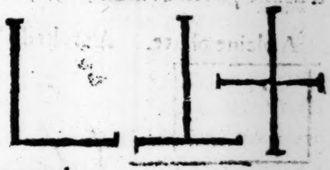
A ſharpe angle is ſo called, becauſe it is leſſer then is a ſquare angle, and the lines that make it, doe not opē ſo wide in their departing, as in a ſquare corner, & if they bée drawne croſſe, all foure corners wil not be equall.

A ſharpe corner.

A blunt or broad corner, is greater then is a ſquare angle, & his line do parte more in ſōder then in a right angle, of which all take the examples.

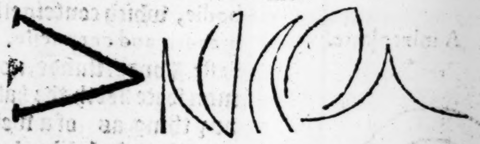
A blunt angle.

Right angles.



And theſe angles (as you ſée) are made partly of ſtreight lines, partlie of crooked lines, and partely of bothe toge‑ther. Howbeit in right

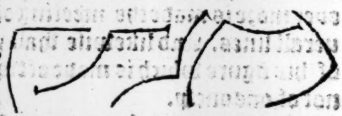
Sharpe angles.



angles I haue put noe example of croked lines, becauſe it

would trouble a lerner to iudge them: for their true iudgemente doeth appertaine to arte perspectiue, and as I may saie, rather to reason then to sense

Blunt or broade angles.



A Plat forme.

But now as of manie prickes there is made one line, so of diuerse lines are there made sundrie fourmes, figures, and shapes, which all yet be called by one proper name, Platte fourmes, and they haue both length and breadth, but yet no deepenesse.

And the boundes of euerie platte forme are lines: as by the examples you may perceiue.

A plaine plat.

Of platte formes some bee plaine and some bee crooked, and some partly plaine, and partly crooked.

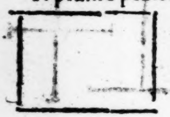
A plaine plat is that which is made all equall in height, so that the middle partes, neither bulke vp, neither shrinke doune more then the both ends.

A crooked plat.

For when the one parte is higher then the other, then is it named a croked plate.

And if it be partly plaine & partly croked, then it is called a Mixte platte, of all which, these are examples.

A plaine platte.



A croked platte

A mixt platte.

And as of many prickes is made a line, and of diuerse lines one Platte forme, so of manie plattes is made a bodie, which conteineth Lengthe, breadth, and deepnesse.

A bodie. Depenesse.

By Deepnesse I vnderstande, not as the cōmon sorte doeth, the hallownesse of any thing, as of a welle, a ditche, a potte, & such like, but I meane the massie thickenesse of any body, as

as in example of a potte: the deepenesse is after the commō name, the space from his brimme to his bottome. But as I take it here, the depenesse of his bodie, is his thickenesse in the sides, which is an other thing cleane different from the depenesse of his holownesse, that the common people meaneth.

Nowe all bodies haue platte formes for their boundes, so in a Die (which is called a cubicke bodie) by Geometriciās and an ashler of Masons, there are sixe sides, which are sixe platte formes and are the formes of the Die.

Cubike.
Ashler.

But a Globe, (which is a body rounde as a boule) there is but one platte fourme, and one bounde, and these are the examples of them both.

A Globe.

A dye or ashler A Globe.

But bicause you shall not muse what I doe call a bound, I meane thereby a generall name, betokening the beginning, ende, and side of any forme.

A bounde.

A forme, figure, or shape, is that thing that is inclosed within one bounde, or many bondes, so that you understand the shape, that the eye doeth discerne and not the substance of the bodie.

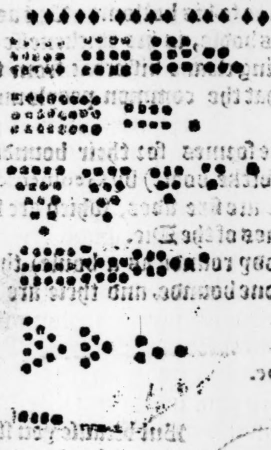
Forme figure.

Of figures there bee many sortes, for either they be made of prickes lines or plat formes. Notwithstanding to speake properly, a figure is made by platte formes, and not of bare lines unclosed neither yet of prickes.

Yet for the lighter forme of teaching, it shall not bee unsemely to call all such shapes, formes and figures, which the eye may discerne distinctly.

And first to begin with prickes, there may bee made diuerse formes of them, as partly here doeth followe.

A linearie



A linearie number

Triangular numbers.

Long square numbers.

Iust square numbers.

A three cornered spire.

A square spire.

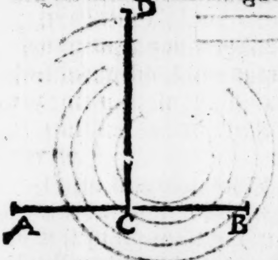
And so may there be infinite formes more, which I omitte for this time, considering that their knowledge appertaineth more to Arithmetike figurall, then to Geometrie.

But yet one name of a pricke, which he taketh rather of his place, then of his forme, may I not ouerpasse. And that is, when a pricke standeth in the middle of a circle (as no circle can be made by compasse without it) then is it called a centre. And therefore doe Masons, and other worke men call that patron a centre, whereby they drawe the lines, for iust hewing of stones for arches, vaulter, and chimneyes, because the chiefe vse of that patron is wrought, by finding that pricke or centre, to which al the lines are drawne, as in the thirde booke it doeth appeare.

A centre

Lines make diuerse figures also, though properly they may not be called figures, as I saied before (vnlesse the lines doe close)

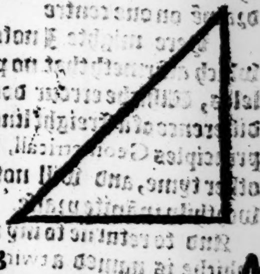
nes doth close (but only for easie manner of teaching, all shall be called figures, that the eye discerneth, of whiche this is one. when one line lieth flat (which is named the grounde line) and an other commeth downe on it and is called a perpendiculare, or plumbe line, as in this example you may see. Where A. B. is the grounde line, and C. D. the plumbe line.



A ground line.

A perpēdiculare line A plumbe line.

And likewaies in this figure there are three lines, the grounde line which is A.B, the plumbe line, that is A. C. and the bias line, which goeth from the one of them to the other, and lieth against the right corner in such a figure which is here C.B.



But considering that I shall haue occasion to declare sundrie figures anon, I will first shew some certaine varieties of lines that close no figures, but are bare lines, and of the other lines will I make mention in the description of the figures.

Paralleles, or Gemowe lines bée suche lines as be drawne forth still in one distaunce, and are no nerer in on place, then in an other, for an if they bée nerer at one ende then at the other, then are they no paralleles, but may be called boughe lines, and loe here examples of them both

Tortuouse paralleles.

Paralleles. Gemowe lines.

I haue

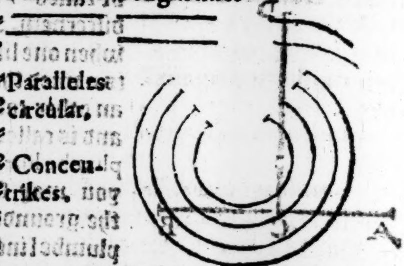
I haue added also paralleles tortuouse, which bowe contrarie waies with their two endes: and paralleles circular, whiche bee like vnperfect cōpasses: For if they bee whole circles, thē are thei called concētriks that is to saie, circles drawē on one centre

Concentrikes

Paralleses, bought lines

Paralleles circular,

Concentrikes.



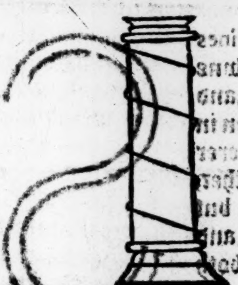
Here mighte I note the errour of good Albert Durer, which affirmeth that no perpendicular lines can bee paralleles, Whiche errour doeth spryng partly of ouersight of the difference of a streight line., and partly of mistakyng certain principles Geometricall, which all I will let passe vntill an other tyme, and will not blame hym, which hath deserued worthily infinite praise.

A twine line

And to returne to my matter, an other fashiō line is there whiche is named a twine or twist line, & it goeth as a wreath about some other bodie. And an other sorte of lines is there, that is called a spirall line, or a worme line, which representeth an apparant forme of many circles, where there is not one in dede: of these two kindes of lines, these be examples.

A spirall line. A worme line

A twiste line.



A spirall line.

A touch line.

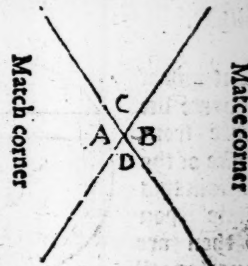
¶ A touche line, is a line that runneth a long by the edge of a circle, onely touching it, but doeth not crosse the circumference of it, as in this example you may see. And when that a line dooeth crosse the edge of the circle, then is it called a cord, as you shall see anon in the speaking of circles.

A corde.

In the meaneseason I must not omitte to declare, what angels be called matche corners, that is to say such as stand directly one against the other, when two lines be drawen a crosse, as here appeareth.

Matche corner.

When A. and B. are match corners, so are C, and D, but not A. & C. neither D, and. A.



Now will I beginne to speake of figures that bee properly so called of which all be made of diuers lines, except only a circle, an egge forme and a tunne forme, which three haue no angle and haue but one line for their bounde, and an eye forme, which is made of one line, and hath an angle only.

A circle.

¶ A circle is a figure made in closed with one line, and hath in the middle of a pricke or centre, from which all the lines that be drawen to the circumference are equall all in length, as here you see.

Circumference,

And the line that encloseth the whole compasse, is called the circumference,

A diametre

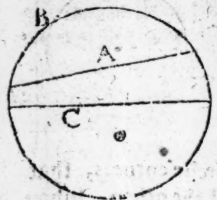
And all the lines that be drawen crosse the circle, and go to the centre are named Diameters, whose halfe I meane from the centre to the circumference

ference any way, and is called the semidiameter, or halfe diameter.

Semidiameter

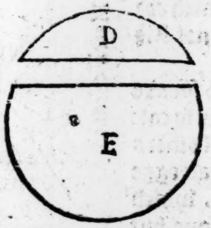
But and if the lines goe crosse circle, and passe beside the centre, then is it called a Corde, or a Stryng line as I said, before, and as this example sheweth: where A. is the corde.

A corde or a string line



And the compassed line that answereth to it, is called an Arche line, or a Bowe line, which here is marked with B. and the Diameter with C.

An arche line A bow line



But and if that parte bee separate from the reste of the circle (as in this example you see) then are both partes called cantelles, the one the greater cantell, as E. and the other the lesser cantell, as D. And if it be parted iuste by the centre (as you see in F.) then is it called a semicircle, or halfe compasse.

A candle.

A semie circle

Somtimes it happeneth that a cantell is cutte out with two lines, drawne from the centre to the circumference (as G, is) and then make it bee called a Nooke cantell, and if it be not parted from the reste of the circle (as you see in H.) then is it called a nooke plainly, without any addition. And the compassed line in it, is called an Arche line, as the example here doeth shewe.

A nooke candle.

A nooke.

An

An arche.

Now haue you heard as touching circles, meetely sufficient instruction, so that it shuld seme nedelesse to speake any more of figures in that kinde, saue that there doeth yet remaine two formes of an imperfecte circle, for it is like a circle that were brused, and therby did runne out ende longe one way, which fourme Geometricians doe call an Egge fourme, because it doeth presente the figure and shape of an Egge duely proportioned (as this figure sheweth) hauing the one ende greater then the other

An Egge forme.

An egge fourme.

A tunne forme.

For if it bee like the figures of a circle pressed in lengthe, and both sides like bigge, then it is called a tunne forme, or barrell fourme, the right making of which figures, I will declare hearafter in the thirde booke.

A tunne or or barrell forme

An other forme there is, which you may call a Nutte fourme and is made of lines, much like an egge forme saue that it hath a sharpe angle.

And it chaunceth sometimes that there is a right line drawen crosse these figures, and that is called an axeline, or axtree. Howbeit, properly that line is called an axetree, which goeth through the middle of a Globe, for as a Diameter is in a circle, so is an axe line or an axetree in a Globe, that line that goeth from side to side, and passeth in the middle

An axetree or axe line.

middle

middle of it. And the two poinctes that such a line maketh in the vtter bounde or platte of a Globe, are named Polis, which you may call aptly in Englishe, tourne poinctes: of which I doe more largely intreate, in the booke that I haue written of the vse of the Globe.

But to retourne to the diuersities of figures that remaine vndeclared the most simple of them are such ones, as be made but of two lines as are the cantle of a circle, and the halfe circle, of which I haue spoken alreadie. Likewise the halfe of an egge fourme the cantle of an egge fourme, the halfe of a tunne fourme, and the cantle of a tunne fourme, and besides these a figure much like a tunne fourme, saue that it is sharpe cornered at both the endes, and therefore doth consiste of two lines, where a tunne fourme is made of on line, and that figure is named an eye fourme.

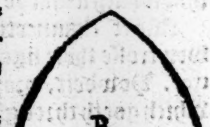
An eye forme

The next kindes of figures are those that be mad of thrée lines, either they be al right lines, al crooked lines either some right, and some crooked. But what fourme soeuer they be of, they are named generally triangles, for a triangle is nothing els to say, but a figure of three corners.

A triangle.

And this is a generall rule looke how many lines any figure hath, so manie corners it hath also, if it be a plat forme and not a bodie. For a bodie hath diuers lines méeting sometimes in one corner.

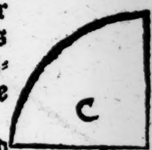
Now to giue you examples of triangles, there is none which is all of crooked lines, and may bée taken for a portion of a Globe, as the figure marked with A.



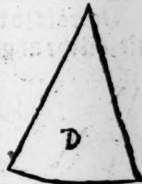
An other hath two compassed lines & one right line, and is as the portion of halfe a Globe, example of B.

An

Another hath but one compassed line, and is the quarter of a circle named a quadrate, and the right lines make a right corner, as you see in C. Other lesse then it as you see D, whose righte lines make a sharpe corner, or greater then a quadrate, as is F. and then the right lines of it doe make a blunt corner.

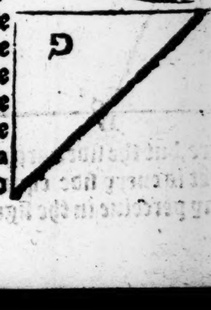


Also some triangles haue all right lines, and they be distincted in sunder by there angles, or corners for either there cornes be all sharpe, as you see in the figure E. either two sharpe and one right square as in the figure G, either two sharpe and one blunt, as in the figure H,



There is also another distinction of the names of triangles, according to there sides, which either bée all equall, as in the figure E. and that the Gréekes doth call Isopleuron, and the Latines æquilaterum, and in Englishe it may bée called a Thelike triangle either els two sides bée equall and the third vnequall, which the Gréekes call Isosceles, the Latines, æquicurio, and in English tweilike may they bée called, as in G. H and K. For they may bée of thrée kindes, that is to sae, with one square angle, as is G. or with a blunte corner as H, or with all in sharpe corners, as you sée in K

Further more it may be that they haue neuer a on side equall to an other & they be in thrée kindes also distincte like the twilikes, as you may perceiue by these examples M. N. and O, where M, hath a right angle, N. a blunt angle and O, al sharpe angles, these the Grekes and the Latines doe call scalena



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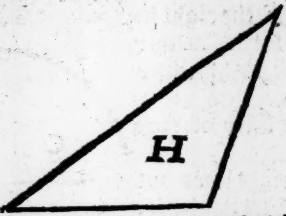
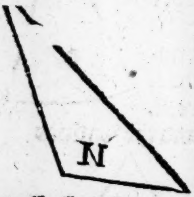


and in English they may be called nouelekes for they haue no side equal or like lõg. to any other in ỹ sãe figure.

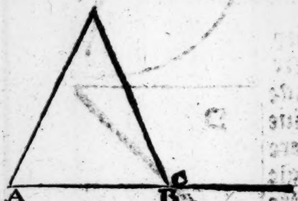
Heere is to bee noted, that in a triangle, all the angles be called innerangles, except any side bee drawen forthe in length, for then is that fowerth corner called an vtter corner, as in this example because A.B. is drawen in length, therefore the angle C. is called an vtter angle.



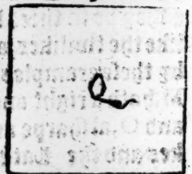
Quadrãgle

And thus haue I doen with triangled figures, and now followeth quadrangles, which are figures of fower corners, and of fower lines also, of which there be diuerse kindes, but chiefly fiue, that is to say, a square quadrate, whose sides bee all equall, & all the angles square, as you se here in this figure Q. The second kind is called a long square whose foure corners be a Square, but the sides are not equall eche to other yet is euery side equal to that other that is against it, as you may perceiue in the figure R.

A square quadrate.

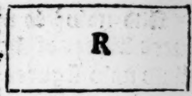


A longe square.



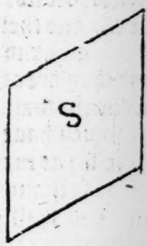
The

The thirde kinde is called Losenges, or Diamondes, whose sides be al equall ,but it hath neuer a square corner, for two of them bee sharpe, and the other two bee blunte, as appeareth in S.



A losenge.
A diamonde.

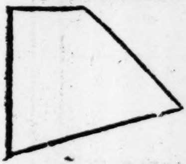
The fowerth sorte are like vnto losenges, saue that thei are longer one waie, & their sides be not equal, yet their corners are like the corners of a losinge, and therfore are thei named Losengelike, or Diamondlike, whose figure is noted with T. Here shall you marke that all those squares, whiche haue their sides all equall, maie bee called also for easie vnderstanding, likesides, as Q. and S. and those that haue onely the contrary sides equall, as R. and T. haue, those will I call likeiammes, for a difference.

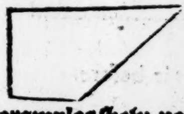


Rhombus.

A losengelike

Rhomboides





The fift sort doeth containe all other fashions of foure cornered figures, and are called of the Greekes Trapezia, of the Latines mensulæ, and of Arabians helmuariphe, they maie bee called in Englishe borde fourmes, they haue no side equall to an other, as these examples shew, neither kepe they any rate in their corners and therefore are they compted vnruled formes, and the other foure kindes onely are compted ruled formes, in the kinde of quadrangles. Of these vnruled formes there is no nomber, they are so many & so diuers, yet by art they maye be chaūged into other kindes of figures, and thereby be brought to measure and proportion, as in the 10. cōclusion is partly taught, but more plainly in my boke of Measuryng you maye see it.

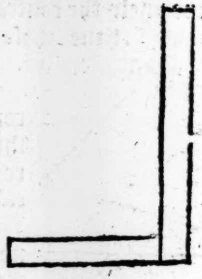
Borde formes

And

And nowe to make an ende of the diuers kindes of figures, there doeth followe now figures of fiue sides, either fiue corners which we may call cink angles, whose sides partly are all equal, as in A. and those are coumpted ruled cinckeangles, and partly vnequal, as in B, and they are called vnruled.

Likewise shall you iudge of siseangles, which haue seuen angles, and so foorthe for as many nombers as there may be of sides, and angles, so many diuerse kindes bee there of figures, vnto which you shall giue names, accordyng to the nomber of their sides and angles, of which for this time, I will make an ende, and I will sette forth one example of a siseangle which I had almost forgotten, and that is it, whose vse commeth often in Geometrie, and is called a Squire, made of two long Squares ioyned together, as in this example sheweth.

A squire.



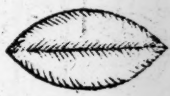
And thus I make and ende to speake of platte fourmes, and will brieflie say some what touching the figures of Bodies which partly haue one platte forme for their bounde, and that iust rounde as a Globe, hath or ended long as is an Egge and a Tunne fourme, whose pictures are these.

The globe as is before.

Howbeit you must marke I meane not the very figure of a Tunne, when I say tūneforme, but a figure like a Tunne, for a Tunne forme

hath but one platte forme, and therefore muste nedes bee rounde at the endes. where as a tunne hath three platte formes, and is flatte at the ende, as partly these pectures doe shewe.

Bodies of two plattes, are either cantles or halues of those others bodies that haue one platte forme, or else they are like in forme to two such cantles ioyned together, as this A. doeth partly expresse: or else it is called a round spire, or stiple forme, as in this figure is some what expressed.



Now of three plattes there are made certaine figures, and bodies as the cantles and halues of al bodies that haue but one platte, and also the halues of halfe globes, & cantles of a globe Likewise a rounde piller, and a spire made of a round spire, slit in two partes long waies.

A round spire.

But as these formes bee hard to be iudged by their pictures so I doe intend to passe them ouer with greate nomber of others formes of bodies which after warde shall be set forthe in the booke of Perspectiue, because that with out perspectiue knowledge, it is not easie to iudge truely the formes of them in flatte protecture.

And thus I make an ende for this time of the definition Geometicall, appertaining to this parte of practise, and the rest will I prosecute as cause shall serue.

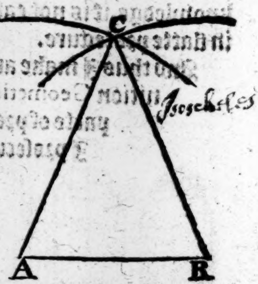
The practike working
of sundrie conclusions Geometricall

The first conclusion
To make a threlike triangle, or any line measueable.

Take the iust length of the line with your compasse, and staie the one foote of the compasse, in one of the endes of that line, turning the other vp or doune at your will, drawing the arch of a circle againste the middle of the line, and doe likewise with the same compasse vnaltered, at the other end of the line, and where these two crooked lines doeth crosse, from thence drawe a line to eche ende of your first line, and therfore appere a threlike triangle, drawen one that line.

Example:

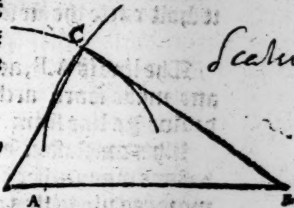
A.B. is the first line on which I would make the threlike triãgle: therfore I open the cõpasse, as wide as ẏ line is long, and draw two arch lines that meete in C. then from C. I drawe two other lines, one to A. an other to B. and then I haue my purpose.



The second conclusion.
If you will make a twilike or a nouelike triangles on any certaine line.

Consider first the length that you will haue the other si-
des

des to containe, and to that length open your comaſſe, and then worke as you did in the thelike triangle, remembring this that in nouelike triangle, you muſt take two lengths beſides the firſt line, & drawe an arche line with one of thẽ at the one ende, the exãple is as ẏ other before.

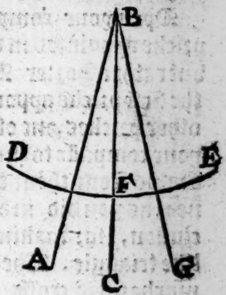


Scalene

The third concluſion.

To diuide an angle of right lines into two equall partes.

Firſt open your compaſſe as largely as you can, ſo that it do not extende the length of the ſhorteſt line that incloſeth the angle. Then ſet one foote of the compaſſe in the very pointe of the angle, and with the other foote drawe a compaſſed arche frõ the one line of the angle to the other, that arche ſhall you deuide in halfe. & then drawe a line from the angles to the middle of the arche, & ſo the angle is diuided into ii. equal partes



Example.

Let the triangles be A.B.C, thẽ ſet I one fote of the compaſſe in B. and with the other I drawe the arche D.E, which I parte into two equall partes in F. and then drawe a line from B. to F. and ſo I haue mine intente.

The fourth concluſion.

To deuide any meaſurable line into two equall partes.

Open youre Compaſſe to the iuſt length of the line. And then ſette one foote ſteddilie at the one ende of the line, and with the other foote drawe an arche of a circle againſt the middle of the line, both ouer it, and alſo vnder it, then doe likewiſe at the other

ende of the line. And marke where those arche lines doe meete crosse waies, and betweene those two prickes draw a line, & it shall cutte the first line in two equall portions.

Example,

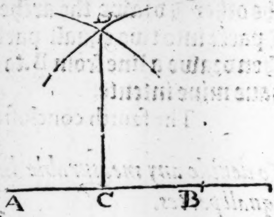
The line is A.B, according to which I open the Compasse and make foure arche lines, which meete in C, and D, then drawe I a line from C, and so haue I my purpose.

This conclusion serueth for making of quadrats & squires, besides many other commodities, howbeit it may bee done more readily to this conclusion that followeth nexte.

The v conclusion.

To make a plumme line, or a pricke that you will in any right line appointed.

Open your compas, so that it be not wider then from the pricke appointed in the line to the shortest ende of the line, but rather shorter. Then set the one foote of the compasse in the first pricke appointed, and with the other foote marke ii. other prickes, one of eche side of that firste, afterward open your compasse to the widenesse of those two newe prickes, & drawe from them two arche lines, as you did in the firste conclusion, for making of a threelike triangle. Then if you doe marke their crossing, and from it drawe a line to your first prick it shall be a iust plumline on that place.



Example.

The line is A.B. the pricke on which I should make the plumme line, is C. then open I the compasse as wide as A·C. and sette one foote in C, with the other doe I marke out C.A, and C.B, then open I the compasse as wide as A.B, and make two arche lines which doe crosse in D, and so haue I done.

How bee it, it happeneth sometimes, that the pricke one

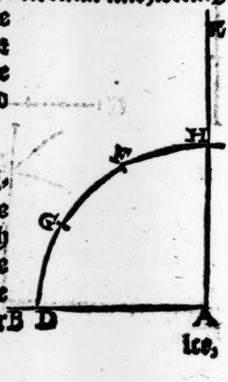
which you would make the perpendicular or plumme line, is so nere the ende of your line, that you can not extende any notable length from it to the one ende of the line, and if so be it then that you may not drawe your line longer from that ende, then doth this conclusion require a newe aide, for the last deuise will not serue. In such case therefore shall you do thus If your line be of any notable length, diuide it into fiue partes And if it be not so longe that it may yeelde fiue notable partes then make an other line at will and parte it into fiue equall portions: so that iij of those partes may be found in your line. Then open your compasse as wide as iij of these fiue measures be, And set the one foote of the compasse in the pricke where you would haue your plumme line to lighte (whch I call the first pricke) & with the other foote drawe an arche line right ouer the pricke, as you can ayme it: then open your compasse as wide as all fiue measures be and set the one foote in the fourth prike, & with the other foote draw an arche line crosse the first, and where they two do crosse thence draw a line to the pointe where you would haue the perpendiculare line to light, and you haue done.

Example.

The line is A·B·and A. is the pricke one. which the perpendiculare line muste light. Therefore I diuide A B. into fiue partes equall, then do I open the compas to the widenesse of three partes (that is A.D: and sette one foote stay in A. and with the other I make an arch line in C. Afterwarde I open the cōpasse as wide as A.B.(that is as wide as all fiue partes) and set one

foote in the fowerth pricke, which is E. drawing an arche line with the other foote in C, also. Then doe I drawe thence a line vnto A, and so haue I doen. But and if the line bee to short to be parted into fiue partes, I shall diuide it into thrée partes onely as you sée the line F.G. and then make D. an other line (as in K. L) which I diuide into fiue such deuisions as. F.G. containeth thrée, then open I the compasse as wide as fower partes (which is K. M) and so set I one foote of the compasse in F, and with the other I drawe an arche line toward H, then open I the compasse as wide as K.L.) that is all fiue partes) and set one foote in G, (that is the iij. pricke) and with the other I drawe an arche line toward H. also, and where those ij arche lines do crosse (which is by H,) thence drawe I a line vnto F, and that maketh a verie plumbe line to F.G. as my desire was. The maner of working of this conclusion, is like vnto the second cōclusiō, but the reasō of it doth depend of the. xlvi. proposicion of the first booke of Euclide

An other way yet set one foote of the compasse in the pricke, on which ye would haue the plumbe line to light, and stretch forth thy other foote towardes the longest ende of the line, as wide as you can for the length of the line & so drawe a quarter of a compasse or more, then without stirring of the compas, set one foote of it in the same line, whereas the circular line did begin, & extende the other in the circular line, setting a marke where it doth light, then take halfe that quantitie more therevnto & by that pricke that endeth the last parte drawe a line to the pricke assigned and it shall be a perpendiculare.



Example.

A.B, is the line appointed, to which I must make a perpendicular line to light in the pricke assigned, which is A. Therefore doe I sette one foote of the compasse in A. and extende the other vnto D. making of a parte of a cir-

cle, more then a quarter, that is D. E Then do I sette one foote of the compasse unaltered in D. and stretch the other in circular line, and it doth light in F, this space betweene D. & F I deuide into halfe in the pricke G, which halfe I take with the compasse, and sette it beyond F, unto H, and therefore is H the point by which the perpendicular line must be drawen, so say I that the line H, A, is a plumbe line to A. B, as the conclusion would.

The vi. conclusion.

To drawe a straght line from any pricke that is not in a line and to make it perpendiculare to an other line.

Open your compasse so wide, that it may extende some what farther, then from the pricke to the line, then set the one foote of the compasse in the pricke and with the other shal you drawe a cōpassed line, that shall crosse that other first line in two places. Now if you deuide that arche line into two equall partes, & from the middle pricke thereof unto the prick without the line, you drawe a straight line, it shalbe a plūbe line to that first line according to the conclusion.

Example.

C. is the apointed pricke. from which unto the line A. B, I minde drawe a perpendicular, Therefore I open the compasse so wide, that it may haue one foote in C. and the other to reache ouer the line, and with that foote I drawe an arch line as you see betweene A. and B. which arch line I deuide in the middle in the pointe D. Then drawe I a line from C. to D, and it is perpendicular to the line A. B, according as my desire was.

The

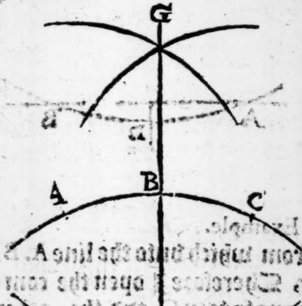
The vii conclusion.

To make a plumbe line or any portion of a circle, and that on the utter or inner bught.

Marke first the pricke where the plumbe line shall light: and pricke out on eche side of it two pointes equallie distante from that first pricke. Then set the one foote of the compas in one of those side prickes, and the other foote in the other side pricke, and first moue one of the feete, and drawe an arche line ouer the middle pricke, then set thy compasse steddie with the one foote in the other side pricke, and with the other foote drawe an other arche line, that shall cut that first arche, and from the very pointe of their meeting, drawe a right line unto the firste pricke, where you doe minde that the plūbe line shall light. And so haue you perfourmed the intente of this conclusion.

Example.

The arche of the circle on which I would erecte a plumbe line, is A.B.C. and B. is the pricke, where I would haue the plumbe line to light. Therefore I meate out two equall distaunces on eche side of that pricke, B, and they are, A. C. Then open I the Compasse as wide as A. C. and setting one of the feete in A. with the other I drawe an Arche line, which goeth by G. Likewises I set one foote of the compas steddily in C. and with the other I draw an arch line, going by G. also. Now considering that G. is the pricke of there meeting it shall be also the point from which you must drawe the plumbe line. Then drawe I a right line from G. to B. and so haue mine intente



Now as A. B. C. hath a plumbe line erected on his vtter bught so may I erect a plumbe line one the inner bught of D.E F, doing with it as I did with the other, that is to say, first setting forth the pricke where the plumbe line shall light, which is E. and then making on other one eche side, as are D, and E. And then procéeding as I did in the example before

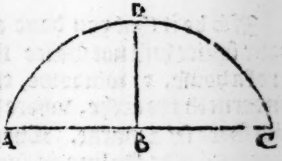
The viii conclusion.

How to deuide the arche of a circle into two equall partes, without measuring the arche.

Deuide the corde of that line into two equall portions, and then from the middle pricke erect a plumbe line, and it shall parte that arche in the middle.

Example.

The arche to be deuided is A D, C. the corde is. A.B.C. this corde is deuided in the middell with B. from which pricke if I erecte a plumbe line as A.B.D, then will it deuide the arche in the middle, that is to say, in D.

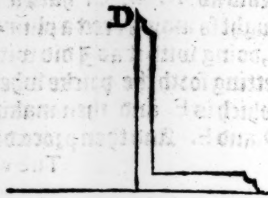


The ix, conclusion.

To doe the same thing otherwise. And for shortnes of worke if you wil make a plūbe line without much labor, you may do it with your squire so that it be iustly made, for if you apply the edge of the squire to the line in which the pricke is, and foresee the very corner of the squire do touch the pricke, And then from that corner, if you drawe a line by the other edge of the squire, it wil be a perpendiculare to the former line.

Example.

A.B. is the line on which I would make the plumbe line or perpendiculare. And therefore I make the pricke, from which the plumbe line must rise, which here is C, Then do I sette one edge of my squire (that is B.C.) to the line A.B. so that the corner of the squire doe touche C. iustly. And from C, I drawe a line by the other edge of the squire (which is C.D.) And so haue I made the plumbe line. D. C, which I sought for.



The x conclusion

How to doe the same thing an other way yet.

If so be it that you haue an arche of suche greatnes, that your squire will not suffice there to as an arche of a bridge, or of a house, or windowe, then may you doe this. Meete vnderneth the arche, where the middle of his corner will be, and there set a marke. Then take a long line with a plummet and holde the line in suche a place of the arch, that the plummet doe hang iustly ouer the middle of the cord, that you did deuide before, and then the line doth shew you the middle of the arche.

Exāple.

The arche is A. D. B. of which I trie the middle thus. I drawe a corde from one side to the other (as here is A. B.) which I deuide in the middle in C. Then take I a line with a plummetie (that is D, E,) and so hould I the line, that the plummet

plummet,E,doth hang ouer C. And then I say that D, is the middle of the arche, And to the intent that my plummet shal point the more iustly, I doe make it sharps in the nether ende and so may I trust this worke for certaine.

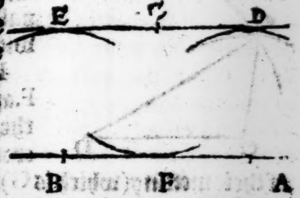
The xj.conclusion.

VVhen any line is appointed, and without a pricke, whereby a parable must be drawen, how you shall doe it.

Take the iust measure betweene the line and the pricke, according to which you shall open your Compasse. Then pricke one foote of your compasse, at the one ende of the line and with the other foote drawe a bowe line, right ouer the pitche of the cõpasse, likewise doe at the other ende of the line, then drawe a line that shall touche the vttermost edge of both those bowe lines, and it will be a true parellele to the first line appointed.

Example.

A,B. is the line vnto which I must drawe an other gemowe line, which must passe by the pricke C. firste I meate with my compasse the smallest distaunce that is from C, to the line, and that is C.F. wherefore staying ỹ Compasse at that distance, I



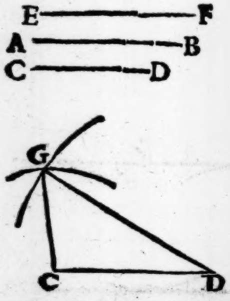
sette one foote in A. and with the other foote I make a bowe line, which is D, then likewise sette I the one foote of the cõpasse in B, and with the other I make the second bowe line, which is E, And then drawe I a line, so that it toucheth the vttermost edge of both these bowe line, and that lines passeth by the pricke C, ande is a gemowe line to A.B. as my seeking was.

The xij.conclusion.

To make a triangle of any three lines, so that the lines be such, that any two of of them be longer then the third. For this rule is generall, that any two sides of euery triangle taken to gether are longer then the other side that remaineth.

If you doe remember the first and the second conclusions, then is there no difficultie in this, for it is in maner the same worke. First consider the thre lines that you may take, and set one of them for the ground line, then worke with the other two lines as you did in the first and second conclusions.

Example.



I haue thre lines. A,B. and C. D, and E.F. of which I putte C.D, for my grounde line, then with my Compasse I take the length of A.B. and sette the one fote of my Compasse in C. and drawe an arche line with the other fote.

Likewise I take the length of E. F. and set one fote in D. and with the other fote I make an arche line crosse the other arche, and the pricke of their meting (which is G) shall be the thirde corner of the triangle. for in all such kindes of working to make a triangle, if you haue on line drawen, then remaineth nothing els but to finde where the pricke of the thirde corner shall be, for two of them must needes be at the two endes of the line that is drawen.

The

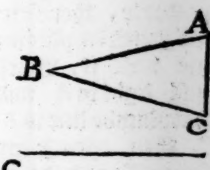
The xiii. conclusion,

If you haue a line appointed, and a pointe in it limited, how you may make on it a lined angle, equall to an other right angle, all redie assigned.

Firste drawe a line against the corner assigned, and so is it a triangle, then take hede to the line, and the pointe in it assigned, and consider if that line from the pricke to this ende bee as longe as any of the sides that make the triangle assigned, and if it be longe enough, then pricke out there the length of one of the lines, and then worke with the other two lines, according to the last conclusion, making a triangle of three like lines, to that assigned triangle. If it be not longe enoughe, then lengthen it firste, & afterwarde doe as I haue saied before.

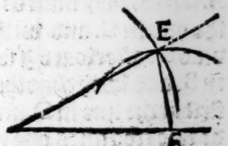
Example.

Lette the angle appointed be A,B,C, and the corner assigned B, Farthermore lette the limitted line bee D.G. and the pricke assigned D,



First therefore by drawing the line A,C, I make the triangle A B, C

Then considering that D. G is longer then A B, you shall cutte out a line from D. towardes G, equall to A.B, as for example D,E. Then measure out the other two lines and worke with them according to the conclusion with the first also and the seconde teacheth you, and then you haue done.

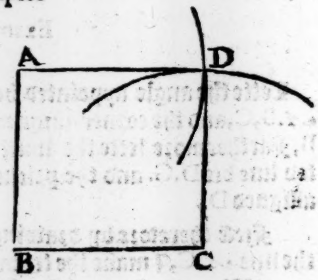


The xiiii conclusion.

To make a square quadrate of any light line appointed

First make a plumbe line vnto your line appointed which shall light at one of the endes of it, according to the fifth conclusion and let it be of like length as your first line is, then open your compasse to the iust length of one of them and sette one foote of the compasse in the ende of the one line, and with the other foote drawe an earch line, there as you thinke that the fowerth corner shall be, after that set the one foote of the same compasse vnsturred in the ende of the other line. & drawe an other arche line crosse the other arche line, and the pointe that they doe crosse in, is the pricke of the fowrth corner of the square quadrate which you seeke for, therefore drawe a line from that pricke to the ende of eche line and you shall ther by haue made a square quadrat.

Examples



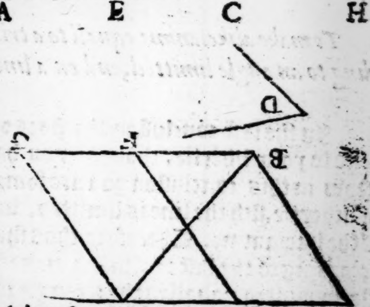
A. B. is the line proposed, of which I shall make a Square quadrate, therefore, first I make a plumbe line vnto it, which shall light in A. and that plumbe line is A. C, then open I my Compasse as wide as the length of A. B. or A.C,) for they must be both equall (and I set the one foote of the end in C. and with the other I make an arche line nigh vn.o D. afterward I set the compasse againe with one foote in B. and with the other foote I make an arche line crosse the first arche line in D, and from the pricke of their crossing, I drawe two lines, one in B, and an other to C, and so haue I made the square quadrat that I intended.

The xv conclusion.

To make a likeiamme equall to a triangle appointed and that in a right lin ed angle limitted.

Firste from one of the angles of the triangle. you shall drawe a gemow line, which shall be a parrallele to that side of the triangle, on which you will make that likeiamme Then on one ende of the side of the triangle, which lieth against the gemowe line, you shall drawe forth a line vnto the gemow line, so that one angle that commeth of those two lines be like to the angle which is limited vnto you. Then shall you deuide into two equall partes, that side of the triangle, which beareth that line and from that pricke of that deuision, you shall raise another line parallele to that former line and continewe it vnto the firste gemowe line, and then of those two last gemowe lines, and the first gemowe lines which is the halfe side of the triangle, is made alikeiamme equall to the triangle appointed. and hath an angle like to an angle limitted according to the conclusion.

Example



B.C.G, is the triangle appointed vnto which I muste make an an equal likeiamme. And, D, is the angle that ẏ likeiãme must haue. Therefore first intending to erecte the likeiãme on the one side that the ground line of the triangle which (is B,G .) I doe drawe a gemow line by C, and make it parallee to the ground line B.G. and that new gemowe line io A.H. Then doe I raise a line from B. vnto the gemowe line (which line is A.B.) and make an angle equall to D, that is the appointed angle (according as the eight cõclusion teacheth, and that angle is B.A.E, Then to procéede, I doe parte in the middle

the

the saied ground line .B,G in the pricke F. from which pricke I drawe to the first gemowe line (A.H.) an other line that is parallele to A.B, and that line is E.F. Now say I that the likeiamme B.A.E.F. is equall to the triangle B.C.G. And also that it hath one angle(that is .B.E, like to D. the angle that was limited. And so haue I mine intente. The profe of the equalnesse of those two figures, doth depende of the xli. proposition of Euclides first boke, and in the. xxxi. propo sition of his seconde boke of Theoremes, which saieth that when a triangle and a likeiamme, bee made betwene two selfe same gemow lines, and haue their ground line of one length, then is the likejamme double to the triangle. whereof it followeth that if two suche figures so drawen, differ in their grounde line onely, so that the grounde line of the likeiamme be but halfe the grounde line of the triangle, then bee those two figures equall, as you shall more at large perceiue by the boke of Theorem es. in the xxxi. Theoreme

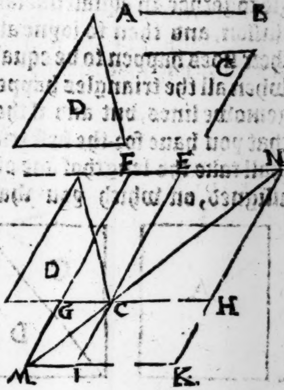
The xvi. conclusion

To make a likeiamme equall to a triangle appointed, according to an angle limitted, and on a line also assigned.

In the last conclusion the sides of your likeiamme were lefte to your libertie, though you had an angle appointed Now in this conclusion you are somewhat more restrained of libertie, sith the line is limitted, which must be the side of the likeiamme. Therefore thus shall you procede. First according to the last conclusion make the likeiamme in the an gle appointed, equall to the triangle that is assigned. Then with your compasse take the length of your line appointed and set out two lines of the same length in the seconde ge- mow lines, be ginning at the one side of the likeiamme, and by these two prickes shall you drawe an other gemowe line, which shall be parallele to two sides of the likeiamme Afterwarde shall you drawe two lines more, for the accom- plishment

plishementes of your workes, which better shall bee perceiued by a shorter example, then by a greater number of wordes, onely without example, therefore by example I will set forth the whole worke.

¶Example.



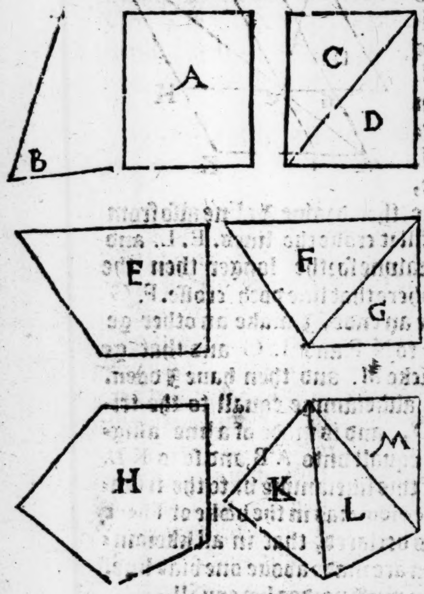
First according to the laste conclusion, I make the likeiamme. E.F.C.G, equall to the triangle D, in the appoincted angle, which is E. Then take I the length of the assined line (which is A B,) and with my compasse I sette forth the same length in the two gemowe lines N.F. and H.G, seting one foote in E, and the other in N. and againe setting on foote in C, and the other in H, After warde I drawe a line from N. to H, which is a gemowe line, to two sides of the likeiamme, then drawe I a line also from N, vnto C. and extende it vntill it crosse the lines. E. L. and F.G, which both must bee drawne forthe longer then the sides of the likeiamme, and where that line doth crosse. F. G there I set M. Nowe to make an ende, I make an other gemow line, which is a parallele to N F. and H. G and that gemow line doth passe by the pricke M. and then haue I doen. Now saie I that H.C.K, L. is alikeiamme equall to the triangle appointed which was D, and is made of a line assigned, that in A.B, for H.C, is equall vnto A B, and so is K.L. The proofe of the equalnes of this likeiamme vnto the triangle, dependeth of the .xxxij Theoreme: as in the booke of Theoremes doeth appeare, where it is declared, that in all likeiammes, when there are more then are made aboue one bias line, the filsquares of euerie of them must needes bee equall.

 ¶The

The.xxij.conclusion.

To make alikeiamme equall to any right lined figure, and that on an angle appointed.

The readiest waie to worke this conclusion, is to tourne that right lined figure into triangles, and then for euery triãgle together an equall likeiamme, according vnto the xi. conclusion, and then to ioyne all those likeiammes into one, if their sides happen to be equall, which thing is euer certaine, when all the triangles happen iustely betweene one paire of gemowe lines, but and if they will not frame so, then after that you haue for the first triangle made his likeiamme, you shall take the length of one of his sides, and set that as a line assigned, on which you shall make the other likeiammes according to the xii. cõclusion and so shal you haue all your likeiames with two sides equall and two like angles, so that you may easilie ioyne them into one figure.

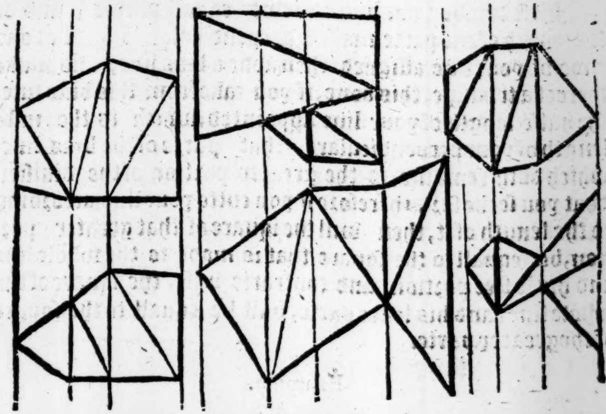


Example.

If the right lined figure bee like vnto A, then may it bee tourned into triangles, that wil stand betwene two paralelles anywaies, as you may see by C, and D, for two sides of both the triangle are paralleles. Also if the right lined figure be like vnto E, thẽ will it bee turned into triangles, lying betwene two paralleles also, as the other did before, as in the example of F.G. But and if the right lined figure bee like vnto H, and so turned into triangles, as

you

you see in K.L.M. where it is parted into .iij. triangles, then will not all those triangles lye betwene one paire of paralleles, or gemowe lines, but must haue many, for euerie triangle must haue one paire of paralleles seuerall, yet it may happen that when there bee three or foure triangles, two of them may happen to agrée to on paire of paralleles which thing I remitte to euery honest witte to serch, for the manner of their draught will declare, how many paire of paralleles they shall neede, of which varietie, because the examples are infinite, I haue set forth these fewe, that by them you may conjecture duely of all other like.



Further explication you shall not greatly nede, if you remember what hath beene taught before, and then diligently beholde, how these sundrie figures be tourned into triangles. In the first you sée I haue made fiue triangles, and fower paralleles, in the seconde seuen triangles, and fower paralleles, in the third three triangles, and fiue paralleles: In the fowth you sée fiue triangles, and fowre paralleles: In the fift, fower triangles, and fower paralleles, and in the sixte there are fiue triangles, and fower paralleles. Howbeit a man may at libertie alter them into diuers formes of triangles

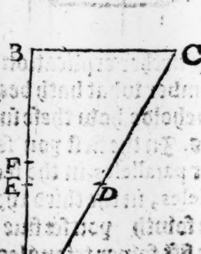
and therefore I leaue it to the descretion of the workemaister to do in all such cases as he shall thinke best, for by these examples (if they be well marked) may all other like conclusions be wrought

¶The.xviij. conclusion

To parte a line aßigned after such a sorte, that the square that is made of the whole line. and of his partes. shalbe equall to the square that commeth of the other parte alone.

First deuide your line into two equall partes, and of the length of one parte make a perpendicular, to light at one ende of your line assigned, then adde a bias line, and make thereof a triangle, this done, if you take from this bias line, the halfe length of your line appointed, which is the iuste length of your perpendiculare, that parte of the bias line, which doth remaine, is the greater portion of the diuision that you seeke for, therefore if you cutte your line, according to the length of it, then will the square of that greater portion, bee equall to the square that is made to the whole line and his lesser portion. And contrarie wise, the square of the whole line, and his lesser parte, will be equall to the square of the greater parte.

Example.



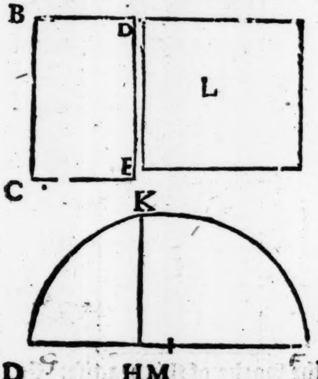
A.B. is the line assigned. E. is the middle pricke of A.B. B.C is the plumbe line or perpendiculare, made of the half of A.B. equall to A.E. either B.E. the bias line is C.A. from which I cut a pece, that is C.D, equall to C.B, and according to the length of the peece that remaineth (which is D.A.) I doe deuide the line A.B. at which deuision I set F. Now say I, that this line A.B. (which was assigned vnto me) is so deuided in this point F that.

that the square of the whole line A.B, and of the one portion (that is F.B, the lesser part) is equall to the square of the other part, which is, F.A, ç, is the greater parte of the first line The profe of this equalitie shall you learne by the.xl. Theoreme.

¶ The.xix. conclusion.

To make a square quadrate equall to any right lined figure appointed,

First make a likeiamme equall to that right lined figure with a right angle, according to the xi. conclusion, then consider the likeiamme, whether it haue all his sides equall, or not: for it they be all equall, then haue you doen your conclusion, but and if the sides be not all equall, then shall you make one right line iust as long as two of those vnequall sides, that line shall you deuide in the middle, and on that pricke drawe half a circle, then cut from that diameter of the halfe circle a certaine portion, equall to the one side of the likeiamme, and from that point of diuision shall you erecte a perpendicular, which shall touche the edge of the circle. And that perpendicular shall be the iust side of the square quadrate, equall both to the like iamme, and also to the right lined figure appointed as in the conclusion willed. ¶Example.



K. is the right lined figure appointed, and B· C. D . E, is the likeiamme, with right angles equall vnto K. but because that this likeiamme is not a square quadrate, I must tourne it into such one after thys sorte, I shall make one right line; as long as two vnequall sides of the likeiamme, that line here is F. G, which is equall to bs B. C, and C.E.

Then part I that line in the middle in the pricke M, and on that pricke I make halfe a circle, acording to the length of the diameter F.G. Afterwarde I cutte away a peece from F. G, equall to C.E marking that pointe with H. And one that pricke I erecte a perpendicular H.K. which is the iust side to the square quadrate that I seke for, therefore according to the doctrine of the tenth conclusion of that line I doe make a square quadrat, and so haue I attained the practise of this conclusion.

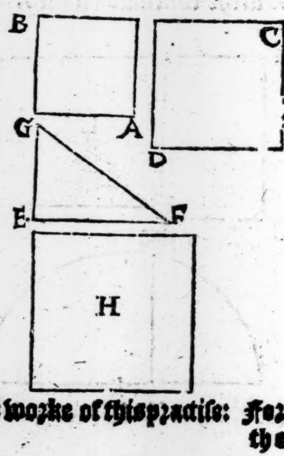
The.xx. conclusion.

VVhen any two square quadrates are set forth, how may you make one square to them both.

First, drawe a right line equall to the side of one of the quadrates: and one the ende of it make a perpendiculare. equall in length to the side of the other quadrate, then drawe a bias line betweene those two lines making thereof a right angled triangle. And that bias line will make a square quadrate, equall to the other two quadrat appointed.

Example.

A.B. and. C.D, are the two square quadrates appointed, vnto which I must make one equal square quadrate. First therefore I doe make a right line E.F, equal to one of the sides of the square quadrate A.B. And on the one end of it I make a plumbe line E. G, equall to the side of the other quadrate D.C, Then drawe I a bias line. G.F, which being made the side of a quadrate (according to the teth conclusion) will accomplishe the worke of this practise: For the

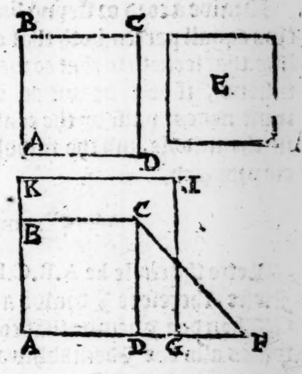


the quadrate H, is as much iust as the other two I meane A. B. and D.C.

The xxj. conclusion.

VVhen any two quadrates bee sette forth, howto make a squire aboute the one quadrate, which shall bee equall to the other quadrate.

Determine with your selfe, aboute which quadrat you will make the Squire, and drawe one side of that quadrate forth in length, according to the measure of the side of the other quadrate, which line you may call the grounde line, and then haue you a righte angle made on this line, by an other side of the same quadrat: Therefore tourne that into a right cornered triangle, according to the worke in the last couclusion, by making of a bias line, and that bias line will perfourme the worke of your desire. For if thou take the length of that bias line with your compasse, and then sette one foote of the Compasse in the farthest angel of the first quadrat (which is the one ende of the grounde line) and extende the other foote on the same line, according to the measure of the bias line, and of that line make a quadrate, enclosing the first quadrate, then will there appeare the forme of a squire aboute the first quadrate, which squire is equal to ẏ second quadrat.



¶Example.

The first square quadrat is A.B.C.D, and the seconde is E. Now would I make a Squire aboute the quadrat A.B.C. D which shall bee equall unto the quadrate E

Therefore

Therefore first I drawe the line A.D, more at length, according to the measure of that side of E, as you see, from D. unto. F. and so the whole line of both these seuerall sides is A.F. then make I a bias line from C. to F, which bias line is the measure of this woorke. Wherefore I open my compasse, according to the length of that bias line C. F, and sette the one Compasse foote in A, and extende the other foote of the compasse towards F. making this pricke G, from which I erecte a plumbe line G.H, and so make out the square quadrate A.G.H.K. whose sides are equall eche of them in A.G. And this square doeth containe the first quadrate. A.B,C.D and also a squire G.H.K, which is equall to the seconde quadrate E, for as the last conclusion declareth, the quadrate A.G.H.K, is equal to both the other quadrates proposed, that is A.B.C.D, and E. Then must the squire G.H.K, nedes be equall to E, considering that all the rest of that great quadrate, is nothing els but the quadrate selfe, A.B.C.D, and so haue I the intente of this conclusion.

¶The xxij conclusion.

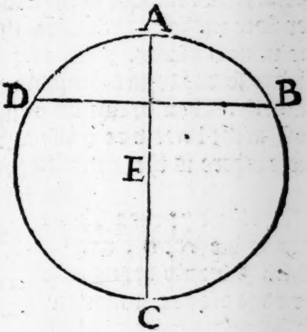
To finde out the centre of any circle aßigned.

Drawe a cord or stryng line crosse the circle, then deuide in two equall partes, both that corde, & also ẏ bow line or arche line, that serueth to that corde, and from the prickes of those deuisions, if you drawe an other line crosse the circle, it must nedes passe by the centre. Therefore diuide that line in the middle, and the middle pricke is the centre of the circle proposed,

¶Example..

Lette the circle be A.B.C.D, whose centre I shall seeke. Firste therefore I drawe a corde crosse the circle, that is A. C. Then doe I deuide that corde in the middle, in E, and likewaies also doe I deuide his arche line A,B C. in the middle, in the poincte B. Afterwarde I drawe a line from B. to E and so crosse the circle, which line B, D. is in which line is

the



the centre that I séeke for. Therefore if I parte ỹ line B.D. in the middle into two equall portions, that middle pricke (which here is F) is ỹ very centre of the saied circle that I séeke. This cõclusion may otherwise bée wroght, as the ỹ most parts of conclusions haue sũdrie formes of practise, and that is by making thrée prickes in the circumference of the circle at libertie wher you will, and then finding the centre of those thrée prickes which worke, because it serueth for sundrie vses, I thinke méete to make it a seuerall conclusion by it selfe.

The. xxiii conclusion.

To finde the common centre belonging to any three prickes appointed, if they be not in an exacte right line.

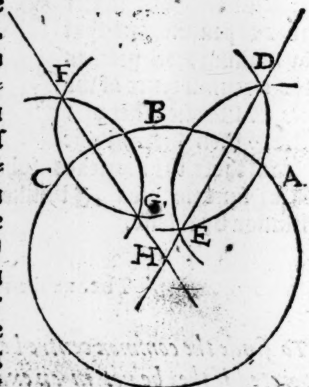
It is to be noted, that though euery small arche of a greater circle doe séeme to bée a right line, yet in the very déede it is not so, for euery parte of the circumference of al circles is compassed, thought in little arches of greate circles, ỹ eye cannot deserne the crokednesse, yet reason doth alwaies declare it, therefore thrée prickes in an exacte right line, can not be brought into the circumference of a circle. But and if they bée not in a right line, howsoeuer they stande, thus shall you finde their common centre. Open your Compasse so wide: that it be somewhat more then the halfe distaunce of two of those prickes, Then set the one foote of the compasse in the one pricke, and with the other foote drawe an

 arches

arche line towarde the other pricke. Then againe putte the foote of your compasse in the seconde pricke, and with the other foote make an arche line, that may crosse the first arche line in two places. Now as you haue doen with those two prickes, so doe with the middle pricke, and the thirde that remaineth. Then drawe two lines by the poinctes, where those arche lines doe crosse, and where those two lines doe meete, there is the centre that you seeke for.

Example.

The three prickes I haue sette to bee A. B. and C. which I would bryng into the edge of one common circle, by findyng a centre common to them all, first therefore I open my compasse, so that they occupie more then the halfe distaunce betwene two prickes (as are A. B.) and so settyng one foote in A. and extendyng the other toward B, I make the arche line D. E. Likewise settyng one foote in B: and turnyng



the other towards A. I drawe an other arch line, that crosseth the firste D, and E. Then from D to E, I drawe a right line D.H. After this I open my compasse to a newe distance and make two arche lines betwene B. and C, which crosse one the other in F and, by which two poinctes I drawe an other line, that is F. H. And because that the line D. H, and the line F.H. doe meete in H, I say that H, is the centre that serueth to those three prickes. Now therefore if you set one foote your compasse in H, and extende the other to any of the three pricke, you may drawe a circle which shall enclose those three prickes in the edge of his circumference, and thus haue you attained the use of this conclusion.

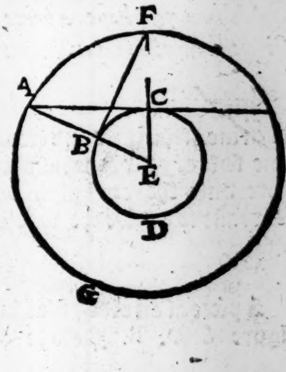
The

The.xxiiij.conclusion.

To drawe a touche line vnto a circle, from any poincte aſsigned.

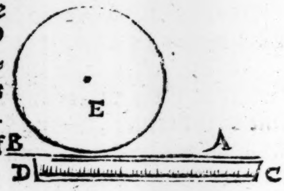
Here muſt you vnderſtand, that the pricke muſt be with out the circle, els the concluſion is not poſſible. But the prick or poinct beyng without the circle, thus ſhall you procede: open your compaſſe, ſo that the one foote of it may bee ſet in the centre of the circle, and the other foote on the pricke appoincted, and ſo drawe an other circle of that largeneſſe about the ſame cētre: and it ſhall gouerne you certainly in makyng the ſaied touch line. For if you drawe a line from the pricke appoincted, vnto the centre of the circle, and marke the place where it doeth croſſe the leſſe circle, and from that poincte erecte a plumbe line, that ſhall touch the edge of the vtter circle, and marke alſo the place wher that plumbe line croſſeth that vtter circle, and from that place drawe an other line to the centre, takyng heede where it croſſeth the leſſer circle, if you drawe a plumbe line from that pricke, vnto the edge of the greater circle, that line I ſay is a toucha line, drawyng from the poincte aſſigned, accordyng to the meanyng of this concluſion.

¶Example.



Lette the circle bee called *B.C. D.* and his centre E, and the pricke aſſigned A, open your Compaſſe now of ſuche wideneſſe, that the one foote may bee ſett in E, which is the Centre of the circle, and the other in A, which is the poincte aſſigned, and ſo make an other greater circle (as here is A.F.G.) then drawe line frō A. vnto E, and where a that line doeth croſſe the inner circle (which here is in

the pricke B.)there erecte a plumbe line unto the line *A.* E. and let that plumbe line touch the utter circle, as it doth here in the pointe *F*, so shall B.F. be that plumbe line. Then from F. unto E, drawe an other line, which shall bee F.E. and it will cutte the inner circle, as it doth here in the pointe C from which pointe C. if you erecte a plumbe line unto A. then is that line A, C. the touch line, which you should finde. Notwithstanding that this is a certaine way to finde any touch line, and a demonstrable fourme, yet more easily manifold may you finde or make any such line with a true ruler, laing the edge of the ruler, to the edge of the circle, and to the pricke and so drawing a righte line, as this example sheweth where the circle is E. the pricke assigned is A. and the ruler C.*D* by which the touch line is drawen, and that is *A.* B. and as this way is light to doe, so is it sertaine enough for any kinde of working.



The xxv conclusion.

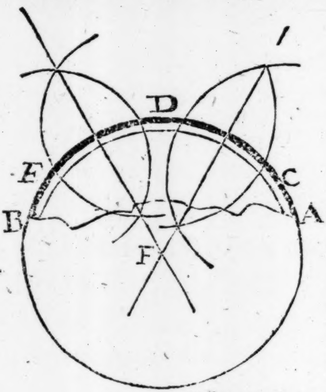
When you haue a peece of a circumference of a circle aßignedhow you may make out the whole circle agreing therunto

First seeke out the centre of that arche, according to the doctrine of the three and twentie conclusion, and then setting one foote of your compasse in the centre extending the other foote unto the edge of the arche, or peece of the circũference it is easie to drawe the whole circle.

Example.

A peece of an olde pillar was founde, like in fourme to this figure *A. D.* B. Now to know how much the compasse of the

the whole piller was, ſeeing by this parte it appeareth that it was rounde, thus ſhall you doe. Make in a table the like draught of the circumference by the ſelfe patron, vſing it as it were a crooked Ruler. Then make three prickes in that arche line, as I haue made C. D, and E. and then finde out the common centre to them all. as the ſeuentene concluſion teacheth. And that centre is here F. now ſetting one foote of your compaſſe in F. and the other in C. D. either in E, and ſo making a compaſſe, you haue your whole intent.



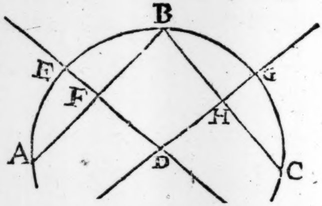
The xxvi. concluſion.

To finde the centre to any arche of a circle.

If ſo be it that you deſire to finde the centre by any other way, then by thoſe three prickes, conſidering that ſometimes you cannot haue ſo much ſpace in the thinge, where the arche is drawen, as ſhould ſerue to make thoſe fower bowe lines, then ſhall you doe thus: Parte that arche line into two partes, eqnall either vnequall, it maketh no force, and vnto eche portion drawe a corde, either a ſtringe line. And then according as you did in one arche in the ſixteneth concluſion, ſo do in both thoſe arches here, that is to ſay, deuide the arche in the middle, and alſo the corde, and drawe then a line by thoſe two diuiſions, ſo then are you ſure that, that line goeth by the centre. Afterwarde do likewiſe with the other arche and his corde, and where thoſe two lines do croſſe, there is the cẽtre that you ſeeke for.

Example.

The arch of the circle A.B.C, vnto which I must seke a centre, therefor first I doe deuide it into two partes, the one of them is A.B and the other is B.C. Then doe I cutte euery arche in the midle, so is E, the middle of A, B, and G, is the middle of B.C. Likewaies, I take the middle of their cordes, which I marke with E, and H, settyng F. by E, and H. by G. Then drawe I a line from E. to F. and frõ G. to H, and they doe crosse in D. wherfore say I, that D. is the centre, that I seke for.



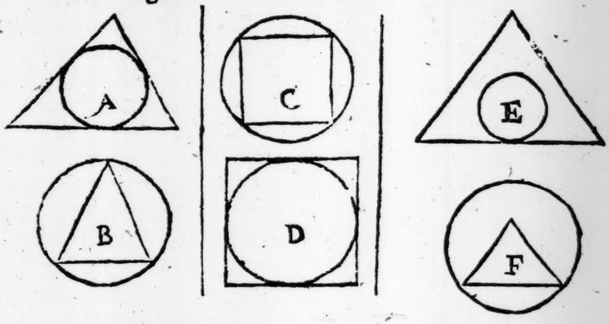
The.xxvii. conclusion.

To drawe a circle within a triangle appoincted.

For this conclusion and all other like, you muste vnderstande, that when one figure is named to bée within an other, that is not otherwaies to bée vnderstande, but that either euery side of the inner figure, doeth touch euery corner of the other, either els euery corner of the one, doeth touch euery side of the other. So I call that triangle drawen in a circle, whose corners doe touche the circumference of the circle. And that circle is contained in a triangle, whose circumference doeth touch iustely euery side of the triangle, and yet doeth not crosse ouer any side of it. And so that quadrate is called properly, to bée drawen in a circle, when all his fower angles doeth touch the edge of the circle. And that circle is drawen in a quadrate, whose circumference doeth touche euery side of the quadrate, and likewaies of other figures.

Examples are these. A.B.C.D.E.F.

A. is the circle in a triangle. C. a quadrate in a circle.



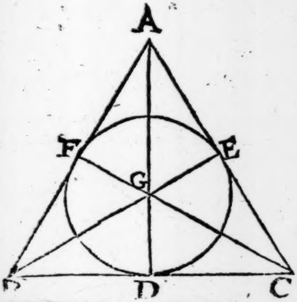
B. a triangle in a circle. D. a circle in a quadrate.

In these two laste figures E. and F, the circle is not named, to bée drawen in a triangle, because it doeth not touch the sides of the triangle, neither is the triangle coumpted to be drawen in the circle, because one of his corners doeth not touch the circumference of the circle, yet (as you sée) the circle is within the triangle, and the triangle within the circle, but neither of thē is properly named to be in the other. Nowe to come to the conclusion. If the triangle haue all thrée sides like, then shall you take the middle of euery side, and frō the contrary corner drawe a righte line vnto that poincte, and where those lines doe crosse one an other, there is the cētre. Then set one foote of the compasse in the centre, and stretch out the other to the middle pricke of any of the sides, and so drawe a compasse, which shall touche euery side of the triangle, but shall not passe without any of them.

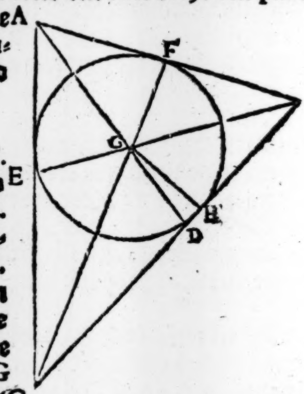
Example.

The triangle is A,B.C, whose sides I doe part into two equall partes, eche by it self in these poinctes D.E.F. putting F. betwene A.B and D. betwene B.C. and E. betwene A.C. Then draw I a line from C. to F. and an other from A. to D

and



& the third from B. to E. And wher al those lines do méete (ẏ is to say M.G.) I set the one fote of my Compasse, because it is the common cētre & so draw a circle according to the distance of any of the sides of the triangle. And thē finde I ẏ circle to agrée iustly to al the sides of the triangle, so that the circle is iustly made in the triangle, as the conclusion did purporte. And this is euer true. when the triangle hath all thrée sides equall, either at the lest two side like longe But in the other kindes of triangles you must deuide euery angle in the middle, as the thirde conclusion teacheth you. And so drawe lines from each angle to their middle pricke. And where those lines doe crosse, there is the common centre, from which you shall drawe a perpendiculare to one of the sides. Then sette one fote of the compasse in that centre, and stretch the other fote according to the length of the perpendiculare. and so drawe your circle.



Exemple.

The triangle is A.B.C. whose corner I haue deuided in the middle with D. E.F. and haue drawen the lines of deuision A. D. B. E. and C. F, which crosse in G. therefore shall G. bée the common centre. Then make I one perpendiculare from G vnto the side A. C, and that

is

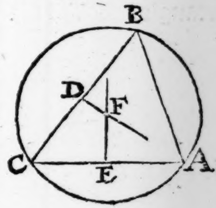
is G.H. Now sette I one foote of the compasse in G. and extende the other foote unto H. and so drawe a cōpasse, which will iustly aunswere to that triangle, accordyng to the meanyng of the conclusion.

The.xxviij.conclusion.

To drawe a circle about any triangle aßigned.

Firste deuide two sides of the triangle equally in halfe, and from those two prickes erecte two perpendiculares, which must nedes meete in crosse, and that poincte of their meetyng is the centre of the circle that muste bee drawen, therefore sette one foote of the compasse in that poincte, and extende the other foote to one corner of the triangle, and so make a circle, and it shall touche all three corners of the triangle.

Example.



A.B.C. is the triangle, whose two sides A.C. and B.C. are deuided into two equall partes in D. and E. settyng D. betwene B. and C. and E. betwene A. and C. And from ech of those two poinctes is there erected a perpendiculare (as you see D.F. and E.F.) which meete, and crosse in F, and stretche forthe the other foote of any corner of the triāgl, and so make a circle, that circle shall touch euery corner of the triangle, and shall enclose the whol e triangle accordyng as the conclusion willeth.

An other waie to doe the same.

And yet an other way may you doe it, accordyng as you

learned in the seuententh conclusion, for if you call the thrée corners of the triangle thrée prickes, and then (as you learned there) if you séeke out the centre to those thrée prickes, and so to make it a circle to inclose those thrée prickes in his circumference, you shall perceiue that the same circle shall iustly include the triangle proposed.

Example.

A.B.C. is the triangle, whose thrée corners I coumpte to bee thrée pointes. Then (as the seuentene conclusion doth teache) I séeke a common centre, on which I may make a circle, that shall incose those thrée prickes that centre. As you sée in D, for in D. doth the right lines, that passe by the angles of the arch lines, méete and crosse. And on that centre as you sée, haue I made a circle, which doth inclose the thrée angles, of the triangle: and consequently the triangle it selfe as the conclusion did intende.

The xxix. conclusion.

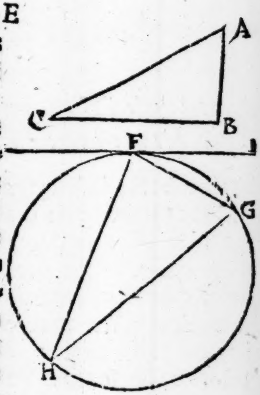
To make a triangle in a circle appointed, whose corners shall be equall to the corners of any triangle assigned.

When I will drawe a triangle in a circle appointed, so that the corners of that triangle, shall be equall to the corners of any triangle assigned. then must I first drawe a touch line vnto that circle, as the twentie iiij conclusion doth teach and in the very pointe of the touch, must I make an angle equall to one angle of the triangle, and that inward toward the circle: Likewise in the same pricke must I make an other angle, with the other halfe of the touch line, equall to an other corner of the triangle appointed, and then betwene

those

thoſe two corners, will there reſulte a thirde angle, equall to the thirde corner of that triangle, Now wherr thoſe two lines that enter into the circle, doe touch the circumferince (beſide the touch line) there ſette I two prickes. & betwéene them I drawe a thirde line. And ſo haue I made a triangle in a circle appointed, whoſe corners be aquall to the corners of the triangle aſſigned.

Example.



A.B.C. is the triangle appointed, and F.G.H. is the circle, in which I muſt make an other Triangle, with like angles, to the angles of A.B. C. the triangle appointed. Therefore firſt I make the touche li D.F.E. And then make I an angle in F. eqtall to A. which is one of the angles of the triangle. And the line that maketh that angle with che touch line is F,H, which I drawe in length vntill it touch the edge of the circle. Then agatne in the ſame pointe F, I make an other corner equall to the the angle C. and the line that maketh that corner with the touch line, is F.G. which alſo I drawe forth vntill it touch the edge of the circle. And then haue I made thrée Angles vpon that one touch line. and in that one pointe F, and thoſe thrée angles bée equall to the thrée angles of the triangles aſſigned. which thing doth plainly appeare, in ſo much as they be equall to two right angles, as you may geſſe by the vi. Theoreme.

 And

And the thrée angles of euery Triangle, are equall also to two right angles as the two and twentie Theoreme doeth shewe, so that because they bée equall to one thirde thyng, they muste néedes bée equall together, as the common sentence saieth. Then doe I drawe a line from G. to H. and that line maketh a triangle F,G.H. whose angles bée equall to the angles of the triangle appoincted. And this triangle is drawen in a circle, as the conclusion did will. The prœfe of this conclusion dœth appeare in the seuentie and fower Theoreme.

The.xxx. conclusion.

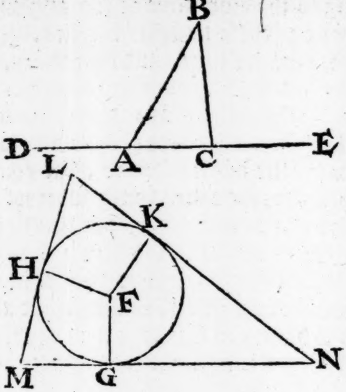
To make a triangle about a circle aßigned, which shall haue corners, equall to the corners of any triangle appoincted.

First drawe forthe in length the one side of the triangle assigned, so that thereby you may haue two vtter angles, vnto which two vtter angles, you shall make two other equall on the centre of the circle proposed, drawyng thrée half diameters from the circumference, which shall enclose those two angles, then drawe thrée touch lines, which shall make two right angles, ech of them with one of those semidiameters. Those thrée lines will make a triangle, equally cornered to the triangle assigned, and that triangle is drawen about a circle appoincted, as the conclusion did will.

Example.

A.B.C. is the triangle assigned, and G.H.K. is the circle appoincted, aboute which I must make a triangle, hauyng equall angles to the angles of that triangle A.B.C. Firste therefore I drawe A.C. (which is one of the sides of the triangle) in length, that there may appeare two vtter angles in that triangle, as you se B.A.D, and B.C.E.

Then



Then drawe I in the circle appoincted a semidiameter, which is here H.F, for F, is the centre of the circle G . H .K. Then make I on that centre an angle equall to the vtter angle *B* . A. D . and that angle is H. F . K. Likewise on the same centre by drawe-yng an other Semidiameter, I make an other angle H. F. G. equall to the seconde vtter angle of the triangle, which isB.C.E.And thus haue I made thrée semidiameters in the circle appoincted Then at the ende of each Semidiameter, I drawe a touch line, which shal make right angles with the semidiameter.And those thrée touch lines méete ,as you sée,and make the triangle L. M. N , which is the triangle that I should make, for it is drawen about a circle assigned, and hath corners equall to the corners of the triangle appoincted, for the corner M.is equall to C.Likewaies L.to A,and N.to *B*,which thyng you shall better perceiue by the sixte Theoreme, as I will declare in the booke of proofes.

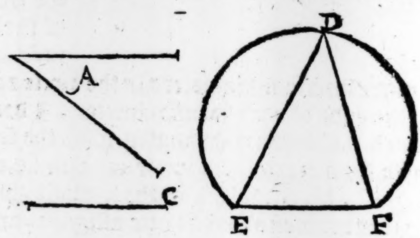
The xxxi.conclusion.

To make a portion of a circle on any right line assigned which shall conteine an angle equall to a right lined angle appoincted

The angle appoincted, may bée a sharpe angle, a righte angle,either a blunt angle, so that the worke must bée di-

 uersly

uersely handeled, according to the diuersities of the angles but considering the hardnesse of those seuerall workes. I will omitte them for a more méeter time, and at this time will shew you one light way, which serueth for all kindes of angles, and that is this. When the line is proposed, and the angle assigned, you shall ioyne that line proposed, so to the other two lines containing the angle assigned, that you shall make a triangle of them, for the easie doing whereof, you may enlarge or shorten as you sée cause, any of the two lines containing the angle appointed. And when you haue made a triangle of those thrée lines, then according to the doctrine of the eight an twentie conclusion, make a circle aboute that triangle, And so haue you wrought the request of this conclusion. Which yet you may worke by the twentie and eighte conclusion also so that of your line appointed you make one side of the triangle be equall to ẏ angle assigned, as your selfe may easily gesse.



Example

First for example of a sharpe Angle, let *A.* stande and B. C. shall bée the line assigned. Then doe I make a triangle, by adding B. C. as a

thirde

thirde side to those other two which doe include the angle assigned, and that triagle is *D*.E.E.so that E.F, is the line appointed, and D. is the assigned. Then doe I drawe a portion of a circle aboute that triangle, from the one ende of that line assigned vnto theother, that is to say, from E. a lōg by D. vnto F. which portion is euermore greater then the halfe of the circle, by reason that the angle is a sharpe angle. But if the angle be right (as in the seconde example you see it) then shall the portion of the circle that containeth that angle, euermore bee the iust halfe of a circle. And when the angle is a blunte angle, as in the thirde example doth propounde, then shall the portion of the circle euermore be lesse then the halfe of a circle. So in the second example, G. is the right angle assigned, and H.K. is the line appointed & L.M.N the portion of the circle aunswering thereto. In the thirde example. O is the blunt corner assigned, P.Q. is the line and R.S.T. is the portion of the circle, that containeth that blunt corner, and is drawen one *R.T.* the line appointed.

The xxxii conclusion

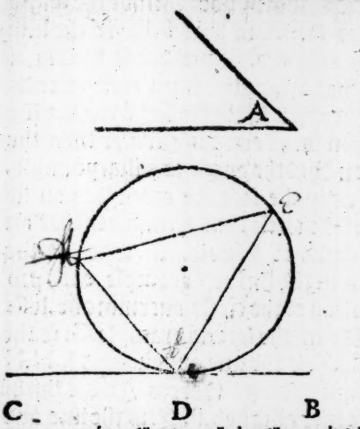
To cutte of from any circle appointed, a portion containing an angle equall to a right lined angle aßigned.

When the angle and the circle are assigned, first drawe a touch line vnto that circle. and then drawe an other line from the pricke of the touching, to one side of the circle, so that thereby those two lines doe make an angle equall to the angle assigned. Then say I that the portion of the circle of the contrarie side of the angle drawen, is the parte that you seeke for.

Example.

A. is the angle appointed, and D, E.F. is the circle assigned from which I must cut a way a portion that doth contain

an



an angle equall to this angle A. Therefore firſt I do drawe a touch line to the circle aſſigned, & that touch line is B, C. the very pricke of the touch is D. from which D. I drawe a line D. E. ſo that the angle made of thoſe two lines be equall to the angle appoincted. Then ſay I, that the arche of the circle D. F. E. is the arche that I ſeke after. For if I do deuide that arch in the middle (as here it is doen in F.) and ſo drawe thence two lines, one to A, and the other to E then will the angle F, be equall to the angle aſſigned

The. xxxiii. concluſion.

To make a ſquare quadrate in a circle aſsigned.

Drawe two diameters in the circle, ſo that they runne a croſſe, and that they make fower right angles. Then draw fower lines, that may ioyne the fower endes of thoſe diameters, one to an other, and then haue you made a ſquare quadrate in the circle appoincted.

Example.

A. B. C. D. is the circle aſſigned, and A. C. and B. D. are the two diameters, which croſſe in the centre E. and make fower right corners. Then doe I make fower other lines, that is A. B, B. C. C. D, and D, A. which doe ioyne together the fower endes of the two diameters, And ſo is

the

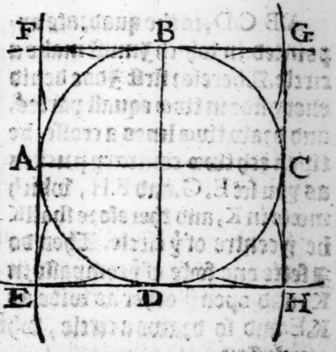
the square quadrate made in the circle assigned, as the conclusion willeth.

The.xxxv.Theoreme.

To make a square quadrate aboute any circle assigned,

Drawe two Diameters in crosse waies, so that they make fower right angles in the centre. Then with youre Compasse take the length of the halfe diameter, and sette one foote of the Compasse, in the eche ende of those diameters drawing two arche lines at euery pitching of the compasse, so shall you haue eight arche lines. Then if you marke the prickes, wherein those arche lines do crosse, and drawe betweene those fower prickes fower right lines, then haue you made the square quadrate, according to the request of the conclusion.

Example.



A.B.C, is the circle assigned, in which first I drawe two Diameters, in crosse waies, making fower right angles, and those two Diameters are *A*.C. and B, *D*. Then sette I my compasse (which is opened, according to the Semidiameter of the said circle (sétting one foote in the ende of euery semidiameter, & drawe with the other foote two arche lines, one on euery side. As first, when I sette the one foote in *A*

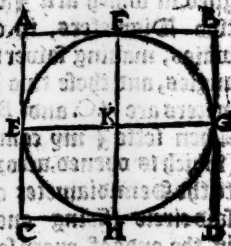
then with the other fote I doe make two arche lines, one in E.and an other in F.Then sette I the one fote of the compasse in B. and drawe two arche lines F and G. Likewise settyng the compasse fote in C. I drawe two other arche lines, G,and H.and on D. I make two other ,H,and E. Then from the crossynges of those eighte arche lines, I drawe fower straigh lines that is to say. E.F.and G. also G,H, and H. E, which fower straight lines doe make the square quadrate that I should drawe aboute the circle assigned.

The,xxxv conclusion

To drawe a circle in any square quadrate appoincted.

First deuide euery side of the quadrate into two equall partes, and so drawe two lines betwene eche two contrary poinctes,and where those two lines doe crosse,there is the centre of the circle. Then sette the one fote of the compasse in that poincte, and stretche forthe the other fote, accordyng to the length of halfe one of those lines,and so make a compasse in the square quadrate assigned.

Example.



A.B.C.D, is the quadrate appointed,in which I must make a circle.Therefor first I doe deuid euery side in two equall partes, and draw two lines a crosse, be twen ech two contrary prickes as you see E. G.and F.H, which meete in K, and therefore shall K be ye centre of ye circle. Then do I sette one fote of ye compasse in K. and open ye other as wide as K.E.and so drawe a circle, which is made accordyng to the conclusion.

The

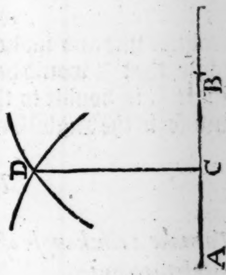
¶The.xxxvj. concluſion.

To drawe a circle aboute a ſquare quadrate.

Drawe two lines betwene fower corners of the quadrate, and where they méete in croſſe, there is the centre of the circle that you ſéeke for. Then ſet one foote of the compaſſe in that centre, and extende the other foote vnto one corner of the quadrate, and ſo maye you drawe a circle, which ſhall iuſtly incloſe the quadrate propoſed.

¶Example.

AB.CD, is the ſquare quadrate propoſed, about which I muſt make a circle. Therefore doe I draw two lines croſſe the ſquare quadrate frõ angle to angle, as you ſée A. C. and B.D. And where they two do croſſe (that is to ſaye in E.) there ſet I the one foote of the compaſſe, as in the centre, and the other foote I doe extende vnto one angl of the quadrat, as for example to A, and ſo make a compaſſe, which doeth iuſtly incloſe the quadrate, accordyng to the mynd of the concluſion.



¶The xxxvij concluſion

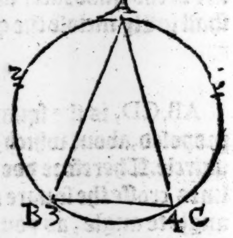
To makge a twileke triangle, whiche ſhall haue euery of the twoo angles that lye about the ground line, double to the other corners.

Firſte make a circle, and deuide the circumference of it into fiue equall partes. And then drawe from one pricke (which you will) two lines to twoo other prickes, that is to ſaye, to the third and fourth pricke, cõptyng that for the firſt wherehence you drewe bothe thoſe lines. Then drawe the third line to make a triangle with thoſe other two, and you haue doen accordyng to the cõcluſion, & haue made a twelike

triangle whose two corners aboute the ground line, are eche of them double to the other corner.

Example.

A.B.C, is the circle, which I haue deuided into fiue equall portions. And from one of the prickes (which is A.) I haue drawen two lines A. B, and B.C. which are drawen to the thirde and fowerth prickes. Then drawe I the thirde line C.B. which is the grounde line and maketh the triangle, that I would haue, for the angle C is double to the angle A and so is the angle B. also.



The xxxviii. conclusion

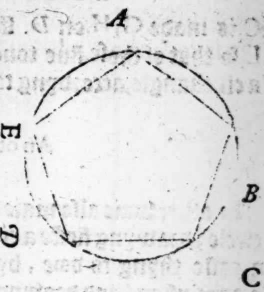
To make a cinckangle of equall sides, and equall corners in any circles appointed.

Deuide the circle appointed, into fiue equall partes, as you did in the last conclusion, and drawe two lines from euery pricke to the other two that are nexte vnto it. And so shall you make a cinckangle, after the meaning of the conclusion.

Example.

You see here this circle *A*.B.C.D.E. deuided into fiue equall portions. And from eche pricke two lines drawen to the other two nexte prickes, so from A. are drawen two sides one to B, and the other to E, and so from C, to B & an other

other to D. and likewise of the reste. So that you haue not onely learned heareby, how to make a sinckeangle in any circle, but also how you shall make a like figure spedily, when and where you will. onely drawe-ing the circle for the intente, readily to make the other figure I meane the cinckeangle) thereby.



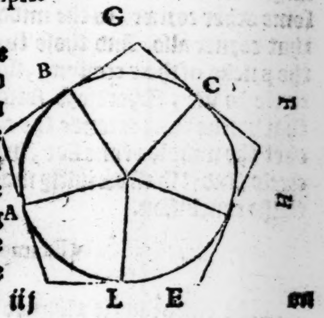
The xxxix. Conclusion.

How to make a cinckangle of equall sides and equall angles aboute any circle appointed.

Deuide first the circle, as you did in the last conclusion in the fiue equall portions, and drawe fiue semidiameters in the circles , Then make fiue touche lines, in such sorte, that euery touche line make two right angles, with on of the semidiameters. And those fiue touche lines. will make a cinckangle of equall sides and equall angles.

Example.

A.B.C,D. E, is the circle appointed, which is deuided into fiue equal partes. And vnto euery pricke is drawen a semidiameter, as you see. Then doe I make a touch line in the pricke B. which is F. G. making two right Angles with the semediameter B. and likewise



onC.is made G, H.on *D*. standeth H,K, and on E, is sette K.L,so that of those fiue touche lines are made the fiue sides of a cinckangle,accordyng to the conclusion.

An otherwaie.

An other waie also maie you drawe a cinckeangle about a circle, drawyng firste a cinckeangle in the circle(which is an easie thyng to doe, by the doctrine of the 8 and thirtie conclusion(and drawyng fiue touche lines, which shall bee iuste paralleles to the fiue sides of the cinckeangle in the circle, foreseyng that [illegible] of them doe not crosse ouertwart an other,and then haue youe doen. The example of this(because it is easie) I leaue to your owne exercise.

¶The.xl.conclusion.

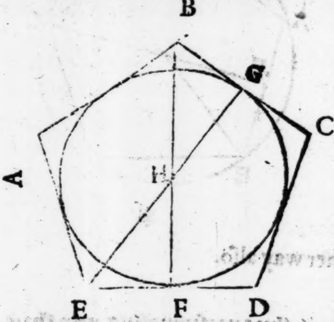
To make a circle in any appoincted cinck eangle of equall sides , and equall corners.

Drawe a plumbe line from any one corner of the cinckeangle, vnto the middle of the side that lieth iust aainst that angle. And doe likewaies in drawyng an other line from some other corner, to the middle of the side that lieth against that corner also. And those two lines will mete in crosse in the pricke of their crossyng, shall you iudge the centre of the circle to bee. Therefore sette one foote of the Compasse in that pricke, and extende the other ende of the line, that touchet the middle of one side, which you liste, and so drawe a circle. And it shall bee iustly made in the cinckeangle, accor to tha conclusion.

¶Example.

The cinckeangle assigned is A.B.C.D. E, in which I must

must make a circle, wherefore I drawe a right line from the one angle (as frō B.) to the middle of the contrary side (which is E.D.) and that middle pricke is F. Then like waies from an other corner (as from E.) I drawe a right line to the middle of the side that lieth againste it (which is B.C.) and that pricke is G. Now because that these two lines doe crosse in H, I saye that H, is the Centre of the circle which I would make. Therefore I set one foote of the compasse in H, and extende the other foote unto G, or F. (which are the endes of the lines that lighte in the middle of the side of that Cinckeangle) and so make I a circle in the cinckeangle, right as the conclusion meaneth.



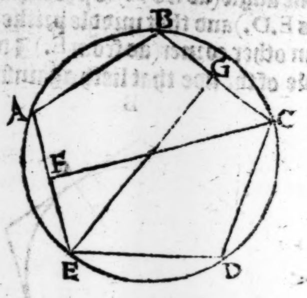
The .xlj. conclusion.

To make a circle aboute any aßigned cinckeangle of equall sides, and equall corners.

Drawe two lines within the cinckeangle, from two corners to the middle, on the two contrary sides (as the last conclusion teacheth) and the poincte of their crossyng shall bée the centre of the circle that I séeke for. Then set I one foote of the compasse in that centre, and the other foote I extende to one of the angles of the cinckeangle, and so drawe a circle about the cinckangle assigned.

Example.

A.B.C.D.E, is the cinckangle assigned, aboute which I would make a circle. Therfore I drawe first of all two lines (as you sée) one from E. to G. and the other frō C, to F. and because



cause they doe mete in H. I say that H. is the centre of the circle that I would haue, wherefore I sette one fote of the Compasse in H. and extende the other to one corner (which happeneth firste (for all are like distaunte from H.) and so make I a circle about the cinkeangle assigned.

An other way also.

An other way may I doe it thus presupposing any thre corners of the cinckeangle, to bee three prickes appointed, unto which I should finde the centre, and then drawing a circle touching them all thre, according to the doctrine of the thre and twentie, and eight and twentie conclusions. And when I haue found the centre, then doe I drawe the circle as the same conclusion doe teache and this fortie conclusion also.

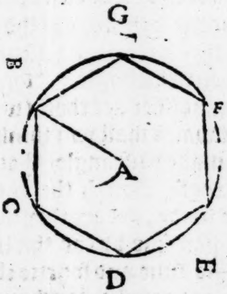
The .xlii conclusion.

To make a siseangle of equall sides, and equall angles, in any circle assigned.

If the centre of the circle bee not knowen, then seke out the centre, according to the doctrine of the twentie conclusion. And with your compasse take the quantitie of the semidiameter iustly. And then sette one fote in one pricke of

the circumference of the circle, and with the other make a marke in the circumference also towarde both sides. Then sette one foote of the compasse stedily in eche of those newe prickes, and poincte out two other prickes. And if you haue doen well, you shall perceiue that there will bee but euen sixe such deuisions in the circumference, whereby it doeth well appere, that the side of any siseangle made in a circle, is equall to the semidiameter of the same circle.

Example.



The circle is B.C.D.E.F.G, whose Centre I finde to bee A. Therefore I sette one foote of the compasse in A, and doe extende the other foote to B, thereby takyng the semidiameter. Then sette I one foote of the compasse vnremoued in B, and mark with the other foote on eche side C. and G. Then from C. I marke D, and from D, E: from E. mark I F. And thus haue I but one space iuste vnto G. and so haue I made a iuste siseangle of equall sides and equall angles, in a circle appointed.

The. xliiii. conclusion.

To make a circle in any siseangle appoincted, of equall sides and equall angles.

¶The.xlv.conclusion.

To make a circle about any siseangle, limited of equall sides and equall angles.

Because you may easily coniecture the makyng of these figures, by that that is saied before of Cinckeangles, onely consideryng that there is a difference in the number of the sides, I thought beste to leaue these vnto your owne deuice, that you should studie in some thinges, to exercise your wit withall and that you might haue the better occasion to perceiue, what difference there is betweene eche two of those conclusions. For though it seme one thyng to make a siseangle in a circle, and to make a circle about a siseangle, yet shall you perceiue, that it is not one thyng, neither are those two conclusions wrought one way. Like waies shall you thinke of those other two conclusion. To make a siseangle about a circle and to make a circle in a siseangle, though the figures be one in fashion, when they are made, yet are they not one in workyng, as you may well perceiue by the thirtie and seuen, thirtie and eight, thirtie and nine, and fourtie conclusions, in which the same workes are taught, touchyng a circle, and a cinckangle: yet this much will I say, for your helpe in workyng, that when you shall seke the centre in a siseangle (whether it be to make a circle in it, either about it) you shall drawe the two crosse lines, from one angle to the other angle that lieth against it, and not to the middle of any side, as you did in the cinckeangle.

The.xlvi.conclusion.

To make a figure of fifteene equall sides and angles in any circle appoincted.

This rule is generall, that howe many sides the figure

shall

shall haue, that shall bée drawen in any circle, into so many partes iustely muste the circle bée deuided. And therefore it is the more easier worke commonly, to drawe a figure in a circle, then to make a circle in an other figure. Now therefore to ende this conclusion, deuide the circle firste into fiue partes, and then ech of them into thrée partes, againe: Or els firste deuide it into thrée partes, and then eche of them into fiue other partes, as you liste, and can moste readily. Then drawe lines betwene euery two prickes that be nighest together, and there will appeare rightly drawen the figure, of fiftene sides, and Angles equall. And so doe with any other figure, of what number of sides so euer it bée.

FINIS.

THE SECOND BOOKE

Of the principles of Geometrie,
containing certaine Theoremes,
which may bee called Approued
truths. And be as it wer the most
certaine groundes, whereon
the practike conclusions
of Geometrie are
founded.

Whereunto are annexed certaine
declarations by examples, for the
right vnderstanding of the same, to
the ende that the simple Reader
might not iustly complaine of hard
nesse or obscuritie, and for the
same cause are the de-
monstrations,
and iust
proofes omitted, vn-
till a more con-
uenient
time.

If truth may trie it selfe
By reasons prudent skill,
If reason may preuaile by right,
And rule the rage of will,
I dare the triall bide,
For truth that I pretende.
And though some liste at me repine.
Iust truth shall me defende.

THE PREFACE
vnto the Theoremes.

I Doubte not gentle reader, but as my argument is straunge and vnacquainted with the vulgare tonge so shall I of many men bee straungilie talked of, and as straungly iudged. Some menne will saye peraduenture, I might haue better imploied my tyme in some pleasaunt histoʀy, compʀisyng matter of chiualry. Some other would moʀe haue pʀailed my trauaile, if I had spente the like tyme in some moʀall matter, either in decidyng some controuersie of Religion. And yet some men (as I iudge) will not mislik this kind of matter, but then will they wish that I had vsed a moʀe certaine oʀder, in placyng both the pʀopositions and Theoremes, and also a moʀe exacter pʀoofe of eche of them both, by demonstrations Mathematicall. Some also will mislike my shoʀtnesse and simple plainesse, as other of other affections diuersely shall espie somewhat that they shall thinke blame woʀthy, and shall misse some what, that they would wish to haue been here vsed. So that euery manne shall giue his verdicte of me accoʀdyng to his phantasie, vnto whom ioinctly, I make this my firste aunswere: that as they are many, and in opinions very diuers, so were it scarse possible to please theim all with any one argument, of what kind so euer it were. And foʀ my seconde answere, I say thus That if any one argumente might please them all then should they be thankfull vnto me foʀ this kinde of matter. Foʀ neither is there any matter moʀe straunge in the English tonge then this, whereof neuer booke was wʀitten befoʀe now, in that tonge, and therefoʀe ought to delight all them, that desire to vnderstande

 strange

straunge matters, as most men commonly doe: And againe the practise is so pleasaunte in vsing, and so profitable in applying, that whosoeuer doth delite in any of both, ought not of right to mislike this arte. And if any man shall like the arte well for it selfe, but shall mislike the fourme that I haue vsed in teaching of it, to him I shall say: First that I doe wish with him that some other man, which coulde better haue done it, had shewed his good will, and vsed his diligence in such sorte, that I might haue béene thereby occasioned iustly to haue lefte of my labour, or after my trauaile to haue suppressed may bokes. But sith no man hath yet attempted the like as farre as I can learne, I trust all such as be not exercised in the studie of Geometrie, shall find greate ease and furtherance by this simple plaine, and easie forme of writing. And shall perceiue the exacte workes of Theon, and others that write on Euclide, a greate deale the soner, by this blunte delineation afore hand to them taught. For I dare presuppose of them, that thing which I haue setin my self, and haue marked in others, that is to say, that it is not easie for a man that shall trauaile in a strange arte to vnderstand at the beginning, both the thinge that is taught, and also the iust reason why it is so. And by experiēce of teaching, I haue tried it to be true, for when I haue taught the proposition, as it imported in meaning, and annexed the demonstration withall, I did perceiue that it was greate trouble, and painful vexation of minde to the learner, to comprehende both those thinges at once. And therefore did I proue first to make them to vnderstande the sence of the propositions, and then afterward did they conceiue the demonstrations much sooner, when they haue the sentence of the propositions, first ingrafted in their mindes. This thing caused me in both these bokes to omitte the demonstrations, and to vse only a plaine forme of declaration, which might best serue for the first introduction. Which example hath béene vsed by other learned men before now, for not onely Georgius Ioachimus Rheticus, but also Boetius that

wittie

wittie clarke,did set forth some whole bookes of Euclide, without any demonstration, or any other declaration at all But and if I shall hereafter perceiue that it may be a thãk-full trauaile,to set forth the propositions of Geometrie,with demonstrations, I will not refuse to doe it,and that with sundrie varieties of demonstrations, both pleasante and profitable also. And then will I in like maner prepare to set, forth the other bookes ,which now are lefte vnprinted,by occation not so much of the charges in cutting of the figures, as for other iust hinderances which I trust hereafter shall be remedied. In the meane season if any man muse why I haue set the Conclusions before the Theoremes,seeing many of the Theoremes seeme to include the cause of some of the conclusions,and therfore ought to haue gone before thẽ as the cause goeth before the effecte. Here vnto I say, that although the cause doe goe before the effecte in order of nature,yet in order of teaching ,the effecte must bee first declared ,and then the cause thereof shewed for so shall men best vnderstand thinges, First to learne that such thinges are to be wrought, & secondarily what they are,and what they doe import,and then thirdly what is the cause thereof. An other cause why that the Theoremes be put after the conclusions is this,when I wrote these first conclusions(which was fower yeares passed) I thought not then to haue added any Theoremes,but next vnto the conclusions to haue taught the order how to haue applied them to worke, for drawing of plattes and such like vses. But after warde considering the greate commoditie that they serue for,and the light that they doe giue to all sortes of practise Geometricall, beside other more notable benefites, which shall be declared more specially in places conuenient, I thought best to giue you some tast of them,and the pleasaunt contemplation of suche Geometricall proportions, which might serue diuersly in other bookes for the demonstration,and proofes of all Geometricall workes. And in them,as well as in the propositiõs I haue drawen in the Linearie examples many times more

ines then be spoken of in the explication of them, which is doen to this intent, that if any man list to learne the demonstrations by harte, as some learned men haue iudged best to doe) those same men should finde the Linearie examples to serue for this purpose, and to want no thyng nedefull to the iuste profe, whereby this boke may be well approued, to be more complete then many men would suppose it.

And thus for this tyme I will make an ende, without any larger declaration of the commodities of this art, or any farther answeryng to that may be obiected against my handelyng of it, willyng them that mislike it, not to meddle with it: and vnto those that will not disdain the studie of it, I promise all such aide as I shalbe able to shewe for their farther proceadyng, bothe of the same, and in all other commodities that therof may ensue. And for their incouragement I haue here annexed the names and brief argumentes of such bokes, as I intende (God willyng) shortly to set forth, if I shall perceiue that my paines may profite other, as my desire is.

The briefe argumentes of such bookes as are appoincted shortly to bee sette forth by the aucthour hereof.

The second parte of Arithmetike, teachyng the workyng by fractions, with extraction of rootes, bothe square and cubike: and declaryng the rule of allegation, with sundrie pleasaunt examples in metalles and other thynges. Also the rule of false position, with diuers examples not onely vulgar, but some appertainyng to the rule of Algeber, applied vnto quantities, partly rationall, and partly surde.

The art of Measuryng by the quadrate Geometricall, and the disorders cōmitted in vsyng the same, not onely reueled but reformed also (as much as to thinstrument pertaineth) by the deuise of a newe quadrat, newely inuented by the aucthour hereof.

The arte of measuryng by the Astronomers staffe, and by the Astronomers ryng, and the forme of making them both.

The art of makyng of Dials, both for the day and the nighte, with certaine newe formes of fixed Dialles for the

Mone

Mone, and other for the sterres, which may be set in glasse windowes, to serue by day & by night. And how you may by those Dials knowe in what degree of the Zodiake, not onely the Sunne, but also the Mone is. And how many howers old she is. And also by the same Diall to know whether any eclipse shalbe that moneth, of the Sunne, or of the Mone.

The makyng and vse of an Instrumente, whereby you may not onely measure the distaunce at once, of all places that you can see together, how much ech one is from you, and euery one from other, but also thereby to drawe the plot of any countrie that you shall come in, as iustely as may be, by mannes diligence and labour.

The vse both of the Globe and the Sphere, and therein also of the art of Nauigation, and what instrumentes serue beste thereunto, and of the true latitude and longitude of regions and townes.

Euclides workes in fower partes, with diuers demonstrations Arithmeticall and Geometricall, or Linearie. The firste parte of platte fourmes. The second of numbers and quantities surde, and irrationall The third of bodies and solide formes. The fowerth of perspectiue, and other thynges thereto annexed.

Beside these I haue other sundry workes, partly ended, and partly to be ended Of the peregrination of man and the originall of all Nations: The state of tymes, and mutacions of realmes: The Image of a perfecte common wealth, with diuers other workes in naturall sciences: Of the wonderfull workes and effectes in beastes, plantes and mineralls, of which at this tyme, I will omitte the argumentes, because they doe appertaine little to this arte, and handle other matters in an other sorte.

To haue, or leaue,
Now maye you chuse.
No paine to please,
will I refuse.

50
19
60
55
180

144
12
288
144
1728

180
282) 1980 (61
1692
288
282
6
1980

The Theoremes of Geometrie,

before which are sette forth certaine
graunctable requestes, which
serue for demonstrations
Mathematicall.

That from any pricke to one other, their may be drawen a right line.

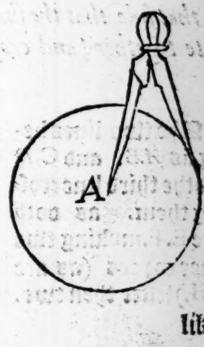
As for exampple *A.*———B.
A being one pricke, and *B.* the other, you may drawe betwéene them, from the one to the other, that is to say from A vnto B. and from *B.* to *A.*

That any right line of measurable length, may be drawen forth longer, and straight.

Example of *A.B*, which as it is a line of measurable length, so may, it be drawen forth farther, as for example vnto C, and that in true straightnesse without crooking.

A *B* C

That vpon any centre, there may be made a circle of any quantitie that a man will:



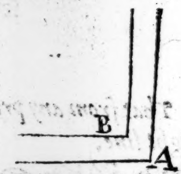
Let the centre bée sette to bée *A.* what shall hender a manne to drawe a circle aboute it of what quantitie that he lusteth, as you sée the forme here: ether bigger or lesse, as it shall

 like

like hym to doe.

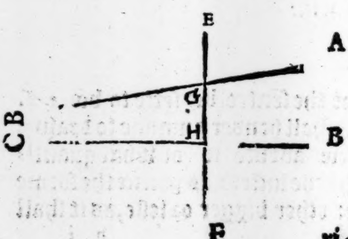
That all right angles bee equall ech to other.

Set for an example A. and B. of whice two though *A.* seme the greater angle, to some men of small experience, it happeneth onely because that the lines aboute A. are longer then the lines about B. as you may proue by drawyng them longer, for so shall B. seeme the greater angle, if you make his lines longer, then the lines that make the angle A. And to proue it by demonstration. I say thus. If any two right corners bee not equall, then one right corner is greater then an other, but that corner which is greater then a right angle, is a blunt corner (by his definition) so must one corner bee both a right corner, and a blunt corner also, which is not possible. And againe: the lesser right corner must bee a sharpe corner, by his definition, because it is lesse then a right angle, which thyng is impossible. Therefore I conclude, that all right angles bee equall.



If one right line doe crosse twoo other right lines, and make twoo inner corners of one side lesser then two right corners, it is certain, that if those ij lines be drawẽ forth right on that side that the sharpe inner corners be they will at lẽgth mete together, and crosse one an other.

The two lines being as *A.*B. and C.*D* and the third line crossing them. as doth here E. F. meking two inner cornes (as are G.H.) lesser then two

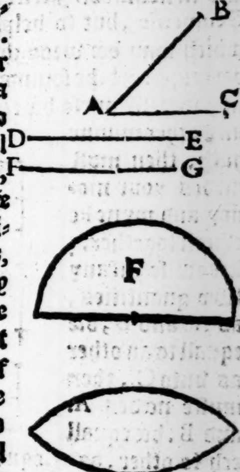


righ

right corners, sith eche of them is lesse then a right corner, as your eyes may iudge, then say I, if those two lines A, B. and C. D. bée drawen in length on that side that G. and A, are, they will at length méete, and crosse one an other.

Two right lines make no platte fourme.

A platte fourme, as you heard before, hath both lengthe and breadth, and is inclosed with lines, as with his boundes, but two right lines cannot inclose all the boundes of any platte fourme. Take for an example, firste these two right lines A. B. and A. C. which méete together in A. but yet cannot bée called a platte fourme, because there is no bonde from B. to C. but if you will drawe a line betwene theim two, that is, from B. to C, then will it bée a platte fourm, that is to say, a triangle, but then are they thrée lines, and not onely two. Likewise may you say of D. E. and F. G. which doe make a platte fourme, neither yet can they make any without helpe of two lines more, whereof the one must bée drawen from D. to F. and the other from E. to G, and then will it bée a long square. So then of two right lines can bée made no platte fourme: But of two croked lines bée made a platte fourme, as you se in the eye fourme. And also of one right line, and one croked line may a platte fourme bée made, as the semicircle F. doeth sette forthe.

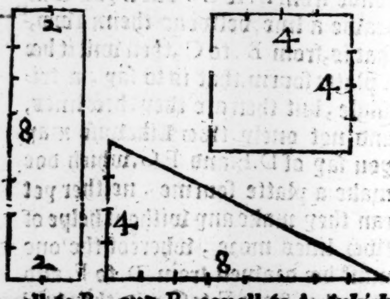


Certaine common sentences manifest to sence, and acknowledged of all menne.

The first common sentence.

VVHat so euer thynges bee equall to one other thing, those same bee equall betwene them selues.

Examples thereof you may take both in greatnes, and also in noumber. Firste (though it pertaine not properly to Geometrie, but to helpe the vnderstandyng of the rules, which may bée wrought by both Artes) thus may you perceiue. If the somme of money in my purse, and the money in your purse bée equal ech of them, to the money that any other manne hath, then must néedes your money and myne be equall together. Likewise, if any twoo quantities, as A. and B, bée equall to an other as vnto C, then muste néedes A. and B. bée equall



ech to other, as A. equall to B. and B. equall to A, which thyng the better to perceiue, tourne these quantities into nomber, so shall A, and B. make sixtene, and C. as many. As you may perceiue by multiplyng the number of their sides together.

The seconde common sentence.

And if you adde equall portions to thynges that be equall what so amounteth of them shall be equall.

Example. If you and I haue like sommes of money, and then receiue eche of vs like sommes more, then our sommes will bée like still. Also if A. and B. (as in the former example) bée equall, then by addyng an equall portion to them both, as to ech of, them the quarter of A. (that is fower) they will bée equall still.

¶The thirde common sentence.

And if you abate euen portions from thynges that are equall, those partes that remaine shall bee equall also.

This you may perceiue by the laste example. For that that was added there, is substracted here. And so thone doeth approue the other.

The fowerth common sentence.

If you abate equalle partes from vnequall thynges, the remainers shall bee vnequall-

As because that a hundreth and eight and fourtie bée vnequall, if I take tenne from them bothe, there will remaine ninetie and eight and thirtie, which are also vnequall. And likewise in quantities it is to bée iudged.

The fiftecommon sentence.

When euen portions are added to vnequall thyn-ges,

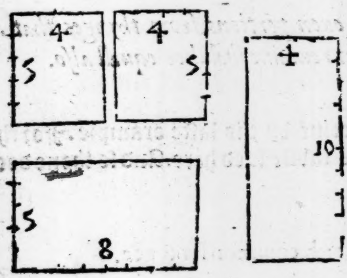
ges, those that amounte shall bee vnequall.

So if you adde twentie to fiftie, and likewaies to ninetie, you shall make seuentie and a hundred and tenne, which are no lesse vnequall then were fiftie and ninetie.

¶The sixte common sentence.

If twoo thynges bee double to any other, those same twoo thynges are equall together.

C D



Because A and B. are eche of theim double to C. therfore must A. and B. nedes be equall together. For as fiue tymes eight maketh fowertie which is double to four tymes fiue, that is xx. so fower times tenne, likewise is double to. xx. for it maketh fowrtie) and therefor must nedes be equall to fowertie.

The seuenth common sentence.

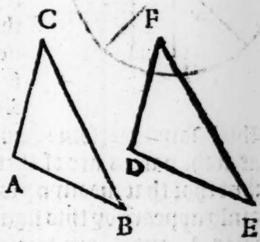
If any twoo thynges bee the halfes of one the other thyng, then are they twoo equall together.

So are D. and C. in the laste example equall together, because they are ech of them the halfe of A. either of B. as their number declareth.

The eight common sentence.

If

If any one quantitie be laied on an other, and they agree, so that the one exceedeth not the other then are they equall together.



As if this figure *A*.B.C, bée layed on that other *D*. E.F. so that A. bée laied to *D.B.* to E, and C. to F. you shall sée them agrée in sides exactly, and the one not to excéede the other, for the line A.B is equall to *D*, E, and the thirde line C.A. is equal to F.D, so that euerie side in the one is equall to some one side of the other, wherefore it is plaine. that the two triangles are equall together.

The nineth common sentences.

Euery thing is greater then any of his partes.

This sentence néedeth none example. For the thing is more plainer then any declaration, considering that other common sentence that follow next that.

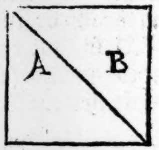
The tenth common sentence.

Euery whole thing is equall to all his partes taken together,

It shall bée méette to expresse both with one example, for this last sentences many menne at the first hearyng doe make a doubte. Therefore, as in this example of the circle deuided into sundrie partes it doeth appere, that no part can bée so great as the whole circle, (according to the meanyng of the eight sentence) so yet it is certaine, that all those eight

partes

partes together bée equall vnto the whole circle. And this is the meanyng of that common Sentence, (which many vse, and fewe dooe rightly vnderstande) that is to say that All the partes of any thyng are nothyng els, but the whole. And cōtrary waies: The whole is nothyng els, but all his partes taken together. Which saiynges some haue vnderstande to meane thus: that al the partes are of the same kinde that the whole thing is: but that that meanyng is false, it dooeth plainly appeare by this figure A.B. whose partes A. and B. are triangles, and the whole figure is a square, and so are they not of one kynd. But and if they apply it to the matter or substaunce of thyngs (as some dooe) then is it most false for every compounde thing is made of partes of diuerse matter and substaunce. Take for example a manne, a house, a booke, and all other compound thynges. Some vnderstande it thus, that the partes all together, can make none other fourme, but that the whole dooeth shew, which is also false, for I may make fiue hundred diuerse figures, of the partes of some one figure, as you shall better perceiue in the thirde booke. And in the meane season, take for an example this square figure folowyng A.B.C.D, which is deuided but into twoo partes, and yet (as you sée) I haue made fiue figures more beside the firste, with onely diuerse ioynyng of those twoo partes. But of this shall I speake more largely in an other place, in the meane season, contente your selfe with these principles, which are certaine of the chief groundes, whereon all demonstrations Mathematicall are fourmed, of which though the moste parte séeme so plaine, that no childe dooeth doubt of them, thinke not therefore that the Arte vnto which they serue, is simple, either childishe, but rather consider, how certaine



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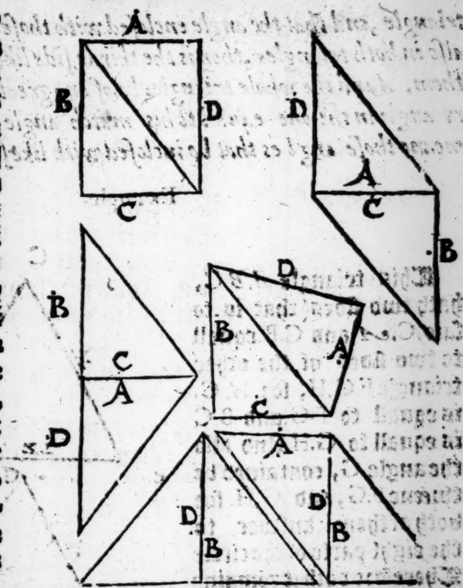


Musicke, and Astronomie) aboue all other artes and scien
ces, that bee vsed amongest men. Thus much haue I saied of the first principles, and now will I goe on with the Theo
remes, which I doe only by examples declare: minding to re
serue the proofes to a peculiar booke, which I will then sette forth, when I perceiue this to bee thankefully taken of the readers of it.

The Theoremes of Geometrie, briefly declared by shorte examples.

The first Theoreme.

WHen two triangles be so drawen, that the one of them hath two sides equall to two sides of the other

triangle ,and that the angle enclosed with those sides be equall also in both triangles ,then is the thirde side likewise equall in them. Aand the whole triangles be of one greatnesse ,and euery angle in the one equall to his match angle in the other, I meane those angl es that be inclosed with like sides,

Example.

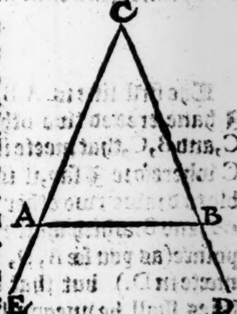
This triangle *A.B.*C, hath two sides(that is to saie)C.*A.*and C.B.equall to two sides of the other triangle F.G.H, for *A.* C. is equall to F·G.and B.C is equall to G.H.And also the angle G, contained betwene F.G, and G,H. for both of them answer to the eight parte of the circle Therefore doth it remaine that *A.*B, which in the thirde line in the first triangle,doth agree in length with F. H. which is the thirde line in the seconde triangle, and the whole triangle *A.B.C.* must neodes bee equall to the whole triangle F,G.H, And euery corner equall to his match, that is to say, *A.* equall to F,B,to H,and C,to G. for those be called match corners, which are inclosed with like sides,either els doe lie against like sides.

The second Theoreme

In twilike triangles the two corners that bee a-

boute the grounde line, are equall together. And if the sides that be equall, be drawen out in length, then will the corners that are vnder the ground line be equall also together.

Example.



A.B.C, is a twiseke triangle, for the one side A.C, is equall to thether side B.C. And therfore I say that the inner corners A, and B, which are about the grounde lines, (that is A.B.) be equall together. And farther if C.A, and C.B, bee drawen forth vnto D. and E, as you see that I haue drawen them, then say I that the two vtter angles vnto A. and B, are equall also together as the Theoreme saied. The proofe whereof, as of all the rest, shall appeare in Euclide, whome I intende to sette forth in English, with sundrie newe additions, if I may perceiue that it will be thankefully taken.

The thirde Theoreme.

If in any triangles there be two angles equal together then shall the sides, that lie against those angles be equall also.

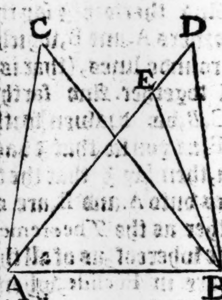
Example.

This triangle A.B.C hath two corners equall eche to other, that is A, and B as I doe by supposition limite, wherefore it foloweth that the side A.C. is equall to that other side B.C, for the side A.C. lieth against the angle B. and the side B.C. lieth against the angle A,

The.iiij.Theoreme.

When two lines are drawen from the endes of any one line and meete in any pointe, it is not possible to drawe two other lines of like length eche to his match, that shall beginne at the same pointes, and ende in any other pointe thē the two first did.

Example.



The first line is A,B on which I haue erected two other lines A C, and B,C, that meete in the prick C wherefore I say, it is not possible to drawe two other line from A, and B, which shall meete in on pointe (as you see A, D, and B, D, meete in D.) but that the match lines shall be vnequall, I meane by matche lines, the two lines on one side, that is the two on the right hand, or the two on the left hande, for as you see in this example A, D, is longer then A,C, and B,C, is longer then B,D. And it is not possible that A,C, and A,D, shall be of one length, if B,D, and B,C, bee like long. For if one couple of eche line bee equall (as the same example A,E, is equall to A, C, in length) then must B,E, needes be vnequall B,C, as you see, it here shorter.

The.v. Theoreme.

If two triangles haue their two sides equall one to an other, and their grounde lines equall also, then shall

shall there corners, which are contained betweene like sides, be equall one the other.

Example.

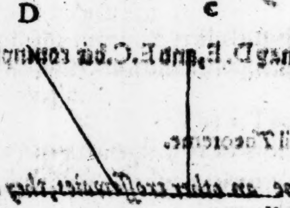
Because these two triangles A.B.C, and D.E.F, haue two sides, equall one to an other. For A,C, is equall to E,F, and againe the ground line A,B, and D,E are like in length, therefore is eche angle of the one triangle equall to eche angle of the other, comparing together

those angles, that are contained within like sides, so is A equall to D. B, to E, and C, to F. for they are contained within like sides, as before is saied.

The vi Theoreme.

VVhen any right line standeth on an other, the two angles that they make, either are both right angles or else equall to two right angles.

Example.



A,B, is right line, and on it there standeth right another right line drawen fro C, perpendicularely on it, therefore say I, that the two angles that they doe make, are two right angles, as may be iudged by the definition of a right angle,

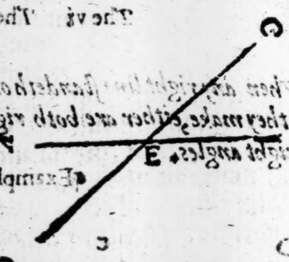
gle. But in the second parte of the example where A. B. be ing still the right line, on which D. standeth in shope waies the two angles that be made of them, are not right angles. but yet they are equall to two right angles for so much as the one is two greate, more then a right angle. so much iust is the other to little, so that both together are equall to two right angles, as you may perceiue.

The. vii. Theoreme.

If two rigt lines be drawen to any pricke in an other line, and those two lines doe make with the first line two right angles, either such as be equall to two right angles, and that towarde one hand, that those two lines do make one straight line.

Example.

A.B. is a straighte line on which their doth light two other lines no from D, and the other from C. but considering that they meate in one pricke E, and that the angles on one hand bée equall to two right corners (as the last Theoreme doth declare) therefore may D. E, and E.C. bée coumpted for one right line.



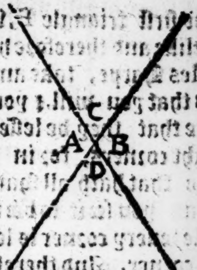
The viii Theoreme.

When two lines do cut one an other crossewaies, they doe make their match angles equall.

Example

¶Example.

What match angles are. I haue tolde you in ỹ definitions of the termes. And here *A*, and *B* are match corners in this examble as art also C. and D, so that the corner A is equall to B. and the angle C. is equall to D.

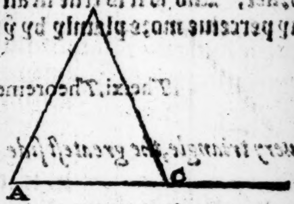


The ix, Theoreme

When so euer any triangle, the line of one side is drawen forth in length, that vtter angle is greater then any of the two inner corners that ioyne not with it.

¶Example.

The triangle A, D, C, hath his ground line A, C, drawen forth in length vnto B. so that the vtter corner that it maketh in C. is greater then any of the two inner corners that lie gainst it, and ioyne not with it, which are A. and *D*. for they both are lesse then a right angle, and be sharp angles, but C. is a blunte angle, and therfore greater then a right angle.

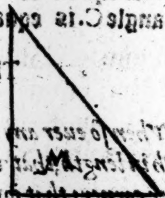


The x, Theoreme.

In euery triangle any two corners, how so euer you take them are lesse then two right corners.

Example.

In the first triangle E, which is a threlike and therefore hath al his angles sharpe, take any two corners that you will, & you shall perceiue that they be lesser then two right corners, for in euery triangle that hath all sharpe corners (as you sée it to bée in this example) euery corner is lesse thē a right corner. And therefore also euery two corners must néedes bée lesse then two right corners. Furthermore in that other triangle marked with M which hath two sharpe corners, and one right, any two of them also are lesse thē two right angles. For though you take the right corner for one, yet the other which is a sharpe corner, is lesse then a right corner, And so it is true in all kindes of triangles, as you may perceiue more plainly by ẏ twentie & two Theoreme.

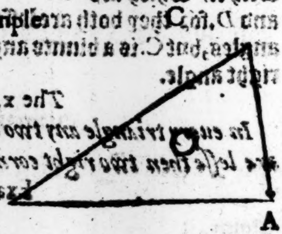


The .xi. Theoreme.

In euery triangle, the greatest side lieth against the greatest angle.

Example.

As in this triangle A.B. C, the greatest angle is C And A,B, (which is the side that lieth against it) is the greatest & longest side, And contrariewise A.C, is the shortest line, so *B.* (which is the ãgle lying against it) in the smallest and sharpest



angle,

angle for this doth follow also, that as the longest side lieth against the greatest angle, so it that followeth.

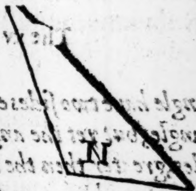
The xii Theoreme

In euery triangle, the greatest angle lieth against the longest side.

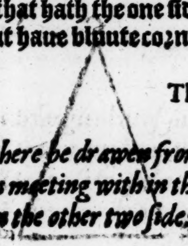
For these two Theoremes are one in truth.

¶The.xxiij. Theoreme.

In euery triangle any two sides together, how so euer you take them are longer then the third.



For example, you shall take this triangle A,B,C, which hath a very blunte corner. and therefore one of his sides greater a good deale. the any of the other, & yet the two lesser sides together are greater then it. And if it be so in a blunte anguled triangle, it must nedes be true in al other for there is no other kinde of triangles that hath the one side so greate aboue the other side, as they that haue blunte corners.



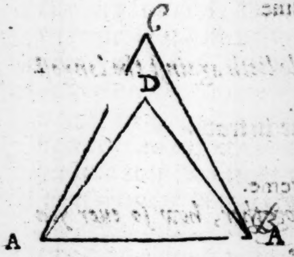
The xiiii. Theoreme,

If there be drawen from the endes of any side of a triangle two lines meeting with in the triangle, those two lines shall bee lesse then the other two sides of the triangle, but yet the corner

 that

of the triangle but yet the corner that they make, shall be greater then that corner of the triangle, whiche standeth over it.

Example.



A.B.C. is a triangle, on whose ground line A.B, there is drawen two lines from the two endes of it, I say from A. and B. and they mete with in the triangles, in the pointe D. wherefore I say that as those two lines A.D. and B.D are lesser then A.C. and B.C, so the angle D. is greater then the angle. C. which is the angle against it

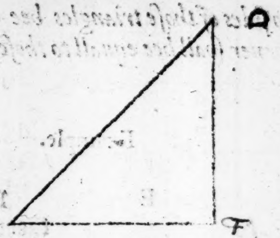
The xv Theoreme.

If a triangle have two sides equall to the the two sides of an other triangle, but yet the angle that is contained betwene those two sides, greater then the like angle in the other triangle, then is his ground line greater then the ground line of the other triangle.

Example.

A.B.C. is a triangle, whose sides A.C, and B.C are equall to E F. and D.E.F. the two sides of the triangle D.E.F. but because the angle in D, is greater then the angle C. which are the two angles contained betwene the equall line, therefore

fore muste the grounde line E. F. nedes be greater then the ground line A. B. as you see plainly.



The xvi. Theoreme.

If a triangle haue two sides equall to to the two side of an other triangle, but yet hath a longer ground line then that other triangle, then is his angle that lieth betweene the equall sides, greater then the like corner in the other triangle.

Example.

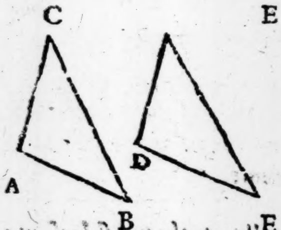
This Theoreme, is nothing els, but the sentence of the laste Theoreme tourned backeward, and therefore neded none other proofe, neither declaration, then the other example

The xvii. Theoreme

If two triangles bee of such sorte, that two angles of the one, be equall to two angles of the other and that one side of the one, be equall to one side of the other, whether that side doe adioyne to one of the equall corners, or els lie against one of them, then shall the

the other two sides of those triangles bee equall together, and the thirde corner shall bee equall to those two triangles.

Example.



Because that A.B.G, the one triangle hath two corners A. and B equall to D. E, that are two corners of the other triangle D. E.F. and that thy haue one side in them both equall, that is A. B, which is equall to D. E. therefore shall both the other two sides bee equall one to another, as A. C. and B·C. equall to D. E, and E.F. and also the thirde angle in them both shall bée equall, that is the angle C. shall be equall to the angle F.

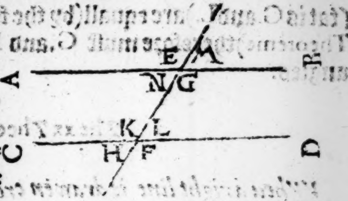
The.xviii.Theoreme.

VVhen on two righte lines there is drawen a third right line crosse waies and make two matche corners of the one line equal to the like two matche corners of the other line, then are those two lines gemowe lines, or paralleles.

Example.

The two first lines are A.B, and C.D the thirde line that crosseth them is E.F, And because that E.F, maketh two match

matche angles with A. B, equall to two other like matche angles on C. D. (that is to say E, G, equall to K. F. and M.N. equall also to H.L.) therfore are these two line A.B. and C.D, gemowe lines understood hereby likematch cornes, those that goe one way, as doth E. G. equall to K.F. likewise N.M. and H.L. for as E.G, and H.L. either N.M, and K.F, goe not one way so be not they like match corners.



The xix Theoreme.

VVhen on two right lines there is drawen a third right line crosse waies, and make the two outer corners toward one hande equall together, then are those two lines. paralleles. And in like manner of two inner corners toward one hand, be equal to two right angles.

Example.

As the Theoreme doeth speake of two outer angles, so must you understand also of two nether angles, for the iudgemente, is like in both. Take for example the figure of the last Theoreme, where A,B. and C.D. bee called paralleles also, because E. and K, (which are two outer corners) are equall, and likewaies L. and M. And so are in like maner the nether corners N, and H. and G, and F. Now to the seconde part of the Theoreme, those two lines A.B, and C,D shall be called paralleles, because these two inner corners As for example, those two that bee towarde the right hande

 (that

(tat is G.and L.)are equall(by the first part of this nineteneth Theoreme)therefore must G.and L be equall to two right angles.

The xx Theoreme.

VVheu a right line is drawen crosse ouer two right gemow lines,it maketh two match corners of the one line,equall to two match corners of the other line, and also both ouer corners of one hand equal together,and both nether corners likewise,and more ouer two inner corners, and two vtter corners also towarde one hand,equall to two right angles.

¶Example.

Because A.B.and C.D.(in the last figure) are paralles,therfore the two match corners of the one line,as E.G, be equall vnto two match corners of the other line, that is K. F, and likewise M.N, equall to H.L. And also E.and K,both ouer corners of the lefte hand equall together,and so are M.and L.the two ouer corners on the right hand,in like manner N. and H the two nether corners on the lefte hande,equall eche to other and G,and F. the two neither angles on the the right hande equall together,

Farthermore,yet G. and L. the two inner angles on the right hande bee equall to two right angles,and so are M.and F. the two vtter angle on the same hand, in like maner shall you say of N. and K. the two inner corners on the lefte hande, and of E.and H. the two vtter corners on the same hand. And thus you see the agreeable sentence of these three Theoremes to tende to this purpose,to declare by the agles how to iudge paralleles & contrary wiaies how you may by paralleles iudge the proportion of the angles.

The

¶The.xxi. Theoreme.

VVhat so euer lines bee paralleles to any other line, those side be paralleles together.

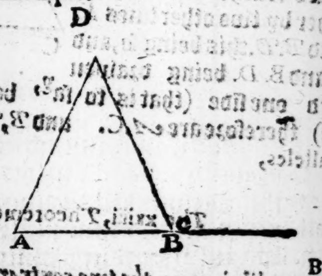
Example.

A.B. is a gemowe line, or a parallele A——B
vnto C.D. And E.F. likewise is a paral- C——D
lele vnto C.D. Wherefore it followeth, E——F
that A.B. must nedes be a parallele vnto E.F.

¶The.xxij. Theoreme.

In euery triangle, when any side is drawen forth in length, the vtter angle is equall to the two inner angle that lie against it. And all three inner angles of any triangles, are equall to two right angles.

Example.



The triangle being A.D. E. and the side A. E. drawen forth vnto B. there is made an vtter corner, which is C, and the vtter corner C. is equall to both the inner corners that lie against it, which are A. and D. and all there inner corners that is to say, A, D, and E. are equall to two right corners, whereof it followeth, that all the three corners of any one triangle, are equall to all the there corners of euerie other triangle. For what so euer things are equall to a-

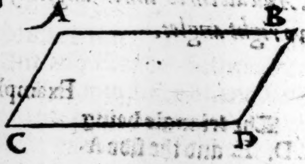
ny

ny one third thing, those same are equall together, by the first common sentence, so that because all the thrée angles of euery triangle, are equall to two right angles, and all right angles bée equall together (by the fourth request) therefore must it néedes follow that all the thrée corners of euery triangle (accompting them together) are equall to thrée corners of a ny other triangle. taken all together,

The x conclusion

When any two right lines doeth touch, and couple two other right lines which are equall in length, and paralleles, and if those two lines be drawen toward on hand, then are they also equall together and paralleles,

Example,



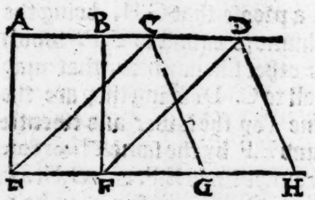
A.B. and C, D. are two right lines. and paralleles, and equall in lengthe, and they are touched and ioyned together by two other lines A. C. and B.D. this being so, and A.C, and B. D. being drawen toward one side (that is to say, both toward the lefte hande) therefore are A.C. and B, D both equall, and also paralleles,

The xxiiii, Theoreme,

In any likeiamme the two contrarie sides are equall together, any so are eche two contrary angles and the bias line that

is drawen in it, doth deuide it into two equall portions.

Example



Here are two likeiammes ioyned together, the one is a longe square *A*.B.E E, & the other is a lösegelike D.C.E.F. which two likeiammes are proued equall togethe, because thyy haue one ground line, that is, F, E. And are made betwene one paire of gemowe lines. I meane A.*D* and E.H. By this *T*heoreme maie you knowe the Arte of the righte measuryng of likeiammes, as in my booke of measuryng, I will more plainly declare.

*T*he.xxvi.*T*heoreme.

All likeiammes that haue equall grounde lines, and are drawen betwene one paire of paralleles, are equall together.

Example.

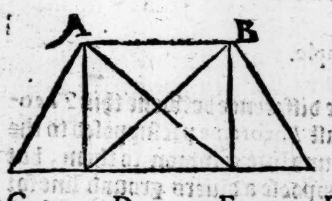
First you must marke the difference betwene this *T*heoreme and the laste, for the last *T*heoreme presupposed to the diuers likeiammes, one ground line common to them, but this Theoreme doeth presuppoose a diuers ground line for euery likeiamme, onely meanyng them to be equall in lẽgth though they bee diuers in number. As for example. In the laste figure there are two paralleles, A.*D*. and E.H. and betwene them are drawen thre likeiammes, the firste is, A. B.E.F: the seconde is E.C.D.F: and the thirde is C.G.H.D

The first and second haue one ground line,(that is E.F)and therefore,in so much as they are betwæne one paire of paralleles,they are equall according to the fiue and twentie Theoreme,but the thirde likeiamme,that is C.G.H.D , hath his ground line G.H. seuerall from the other,but yet equall vnto it. Wherefore the thirde likeiamme, is equall to the other two first likeiammes. And for a proofe that G.H, being the ground line of the thirde likeiamme is equall to E.F, which is the ground line to both the other likeiammes, that may be thus declared G.H, is equall to C. D. sæing they are the contrary sides of one likeiamme (by the fower and twentie Theoreme) and so are G.D.and E.F by the same Theoreme Therefore,sæing both those grounde lines E.F. and G,H, are equall to one thirde line(that is C·D· they must næedes be equall together by the first common sentence.

The xxvi Theoreme.

All triangles hauing one ground line, and standing betweene one paire of paralleles, are equall together

Example.



A.B. and C.F. are two gemowe lines, betweene which there bæ made two triangles, *A.* D. E, and *D*.E.B. so that D.E. is the common grounde line to them both, wherfore it doth followe, that those two triangles A.D·E. and *D*.E.B. are equall eche to other

All triangles that haue like long grounde lines, and bee made betwene one paire of gemowe lines, are equall together.

Example.

Example of this Theoreme, you may see in the laste figure, whereas sixe triangles made betwene those two gemowe lines A.B. and C.F, the firste triangle is A.C.D, the seconde is A.D.E: the third is A.D.B: the fowerth is A.B.E. the fifte is D.E.B. the sixte is B.E.F. of which sixe triangles A.D.E. and D.E.B. are equall, because they haue one common ground line. And so likewise A.B.E. and A.B.D. whose common ground line is A.B. but A.C.D is equall to B.E.F beyng both betwene one couple of paralleles, not because they haue one ground line, but because they haue their grounde lines equall, for C.D. is equall to E.F. as you may declare thus. C.D. is equall to A.B. (by the fower and twentie Theoreme) for they are two contrary sides of one likesiamme. A.C.D. and E.F. by the same Theoreme, is equall to L.F. likewise the triangle A.C.D, is equal to A.B.E. because they are made betwene one paire of paralleles, and haue their ground lines like, I meane C.D. and A.B. Againe A.D.E. is equall to eche of them bothe, for his ground line D.E, is equall to A.B. in so much as they are the contrary sides of one likeiame, that is the long square A.B.D.E, And thus may you proue the equalnesse of all the reste.

The.xxjx Theoreme.

All equall triangles that are made on one ground line, and rise one waie, muste needes bee betwene one paire of paralleles.

Example.

Take for example A.D.E, and D.E.B. which (as the twentie and seuen conclusion doeth proue) are equall together, and as you see, they haue one grounde line D.E. And againe they rise towarde one side, that is to saie, vpwarde towarde the line A.B, wherefore they muste nedes be inclosed betwene one paire of Paralleles, which are here in this example A.B. and D.E.

Example.

Equall triangles that haue their grounde lines equall, and be drawen towarde one side, are made betweene one paire of paralleles.

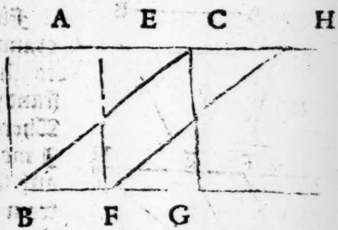
¶Example.

The example that declareth the laste Theoreme, maie well serue to the declaration of this also. For those two Theoremes doe differ but in one poincte; that the Theoreme meaneth of triangles, that haue one grounde line common to them both, and this Theoreme doeth presuppose the ground lines to be diuers, but yet of one length, as A.C.D. and B.E.F, as they are two equal triangles approued by the eight and twentie Theoreme, so in the same Theoreme it is declared, that their grounde lines are equall together, that is C.D. and E.F, now this beyng true, and consideryng that thei are made towarde one side, it followeth, that thei are made betwene one paire of paralles, when I saie, drawen toward one side, I meane that the triangles must be drawẽ either bothe vpward frõ one parallele, either els bothe down ward, for if the one be drawen vpward, and the other down warde, then are they drawen betwene two paire of paralleles, presupposyng one to be drawen by their grounde line, and then doe they rise toward contrary sides.

The.xxx.j Theoreme.

If a likeiamme haue one grounde line with a triangle, and be drawen betwene one paire of paralleles, then shall the like iamme be double to the triangle.

Example.



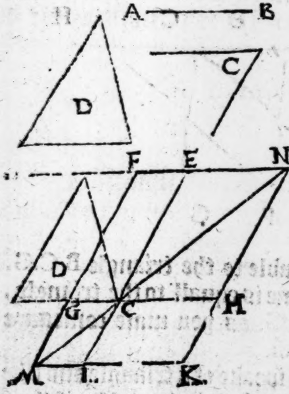
A.H and B.G. are two gemowe lines, betwene which there is made a triãgle B.C.G and a likeiamme A.B.G.C, which haue a ground line that is to saie, B.G. Therefore doeth it followe, that the likelamme A.B.G.C, is double to the triangle B.C.G. For euery halfe of that likeiamme is equall to the triangle, I meane A.B.F.E. either F.E.C.G. as you maie coniecture by the.xj. conclusion Geometricall.

And as this Theoreme doeth speake of a triangle and like iamme, that haue one ground line, so it is true also, if their ground lines bee equall, though they bee diuers, so that they bee made betwene one paire of paralleles. And hereof maie you perceiue the reason, why in measuryng the platte of a triangle, you muste multiplie the perpendicular line by half the ground line, or els the whole ground line by halfe the perpendicular, for by any of these bothe waies, is there made a likeiamme equall to halfe such a one, as should bee made on the same whole ground line with the triangle, and betwene one paire of paralleles Therefore as that likeiamme is double to the triangle so the halfe of it, muste neede bee equall to the triangle.. Compare the eleuenth conclusion with this Theoreme.

The xxxii The oreme.

In all likeiammes, where there are more then one made aboute one bias line, the fill squares of euery of them must nee des bee equall.

Example.



First, before I declare the examples, it shall bée méete to shewe the true understandyng of this Theoreme. Therefore by the Bias line, I meane that line, which in any square figure doeth run from corner to corner. And euery square which is deuided by that bias line, into equall halfes from corner to corner (that is to say, into two equall triangles) those bée compted to stande about one bias line, and the other squares, which touch that bias line, with one of their corners onely, those doe I call Fill squares, accordyng to the Greeke name, which is anapleromata, and called in Latine supplementa, because they make one generall square, includyng and enclosyng the other diuers squares, as in this exãple H.C.E.N. is one square likeiamme, and L.M.G.C. is an other, which both are made aboute on bias line, that is, N.M. then K.L.H.C. and C.E.F.G. are two fill squares, for they doe fill up the sides of the two first square like iammes, in such sorte, that of all them fower is made one greate generall square K.M.F.N.

Now to the sentence of the Theoreme, I saie, that the two

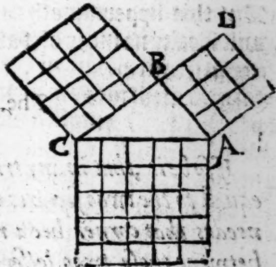
two fill squares.H.K.L.C.and C.E,F.G, are both equall to gether(as it shall be declared in the booke of profes) because they are the fill squares of two likeiammes,made aboute on bias line as the example sheweth.Conferre the twelfth conclusion with the Theoreme.

The xxxii.Theoreme

In all right angeled triangles the square of that side,which lieth against the right angle, is equall to the two squares of both the other sides.

Example.

A,B,C. is a triangle, hauing a right Angle in. B. Wherefore it followeth that the square of A.C. (which is the side that lieth against the right angle) shall be as much as the two squares of A.B. and B.C. which are the other two sides.



By the square of any line you must vnderstande a figure made,iust square,hauing all his fower sides equall to that line. whereof it is the square,so is A..C.F, the square of A.C, Likewise A.B.*D*. is the square of A.B And B.C.E. is the square of B.C.Nowe by the nomber of the diuisions in eche of these squares, may you perceiue not onely what the square of any line is called,but also that the *T*heoreme is true,and expressed plainly both by lines and number. For as you see,the greater square(that is A.C.F) hath v. deuisiōs on eche side,all equall together,and those in the whole square are

are .xxv. Now in the left square, which is A.B.D. there are but three of those deuisions in one side, and that yeldeth nine in the whole. So likewaies you see in the meane square A.C.E. in euery side fower partes, which in the whole amoũt vnto sixtene. Now adde together all the partes of the two lesser squares, that is to saye, sixtene and nine, and you perceiue that they make twentie and fiue, which is an equall number to the somme of the greater square

By this Theoreme you maye vnderstand a readie waie, to knowe the side of any right angled triangle that is vnknowen, so that you knowe the length of any two sides of it. For by tournyng the two sides certaine into their squares, and so addyng them together, either subtracting the one from the other (accordyng as the vse of these Theoremes I haue set forth) and then findyng the roote of the square that remaineth, which roote (I meane the side of the square) is the iuste length of the vnknowen side, which is soughtfor. But this appertaineth to the thirde booke, and therefore I will speake no more of it at this tyme.

The.xxiiii. Theoreme.

If so be it, that in any triangle, the square of the one side, bee equall to the twoo squares of the other twoo sides, then muste needes that corner bee a right corner, whiche is contained betwene those twoo lesser sides.

Example.

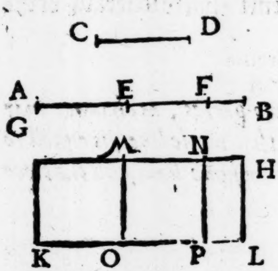
As in the figure of the last Theoreme, because A.C. made in square, is as much as the square of A.B. and also as the square of B.C. ioyned bothe together, therfore the angle that is inclosed betwene those two lesser lines, A.B. and B.C. (that is to saie) the angle B. which lieth against the line A.C. must nedes be a right angle. This Theoreme doeth so depẽde of the truthe of the laste, that when you perceiue the truth

of the one, you can not iustly doubte of the others truthes, for they containe one sentence, contrarie waies pronounced.

The .lxvi. Theoreme.

If there be sette forth two right lines, and one of them parted into sundrie partes, how many or few so euer they be, the square that is made of those two right lines proposed, is equall to all the squares, that are made of the vndeuided line, & euery part of the deuided line.

Example.



The two lines proposed, are A.B. and C.D. and the line A.B. is deuided into thre partes by E, and F. Now saith this Theoreme, that ye square that is made of those ii whole lines A.B. and C. D, so that the line A.B. standeth for the length of the square, and the other line C, D, for the bredth of the same. That square (I say) will be equall to al the squares that be made, of the vndeuided line. (which is C,D.) and euery portion of the deuided line And to declare that particularly: First I make an other line G,K, equall to the line C,D, and the line G.H. to be equall to the line A.B, and to be deuided into thre, like partes, so that G, M. is equall to A.E and M,N, equall to E,F, and then must N,H, nedes remaine equall to F.B, Then of those two lines G,K. vndeuided & G.H. which is deuided I make a square, that is G,H,K,L. In which square if I drawe crosse lines from one side to the other, according to the deuisions of the line G,A, then will it

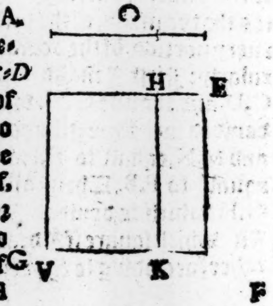
appere plain, that the Theoreme doeth affirme. For the first square G.M.O.K, must nedes be equall to the square of the line C. D, and the first portion of the deuided line, which is A.E, for because their sides are equall. And so the seconde square that is M.N.P.O. shal be equall to the square of C.D. and the second part of A.B, that is E.F. Also the third square which is N.H.L.P, must of necessitie be equal to the square of C.D, and F.B, because those lines be so coupled that euery couple are equall in the seuerall figures. And so shall you not onely in this example, but in al other finde it true, that if one line bee deuided into sondrie partes, and an other line whole and vndeuided, matched with hym in a square, that square which is made of these twoo whole lines, is as much iuste and equally as all the seuerall squares, which bée made of the whole line vndeuided, and euery parte seuerally of the deuided line.

The.xxxvj. Theoreme

If a right line be parted into two partes, as chaunce may happe, the square that is made of that whole line, is equall to both the squares that are made of the same line, and the twoo partes of it seuerally.

Example.

The line propounded beyng A. B. and deuided, as chaunce happeneth in C. into twoo vnequall partes, I say that the square made of the whole line A. B. is equall to the twoo squares made of the same line with the twoo partes of it self, as with A. C. and with C. B, for the square D. E. F. G. is equall to the twoo other partiall squares of D,H

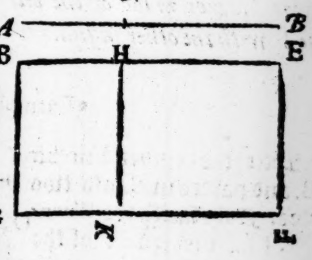


D.H.K.G. and H.E.F.K, but that the greater square is equal to the square of the whole line A.B. & the partial squares equall to the squares of the second partes of the same line, ioyned with the whole line, your eye may iudge without much declaration, so that I shall not nede to make more exposition therof, but that you may examine it, as you did in the laste Theoreme.

The xxxvii Theoreme.

If a right line bee deuided by chaunce, as it maie happen, the square that is made of the whole line, and one of the partes of it, which so euer it bee, shall bee equall to that square that is made of the two partes ioyned together, and to an other square made of that parte, which was before ioyned with the whol line

Example.



The line A.B. is deuided in C. into two partes, though not equally, of which two partes, for an example I take the firste, that is A.C. and of it I make one side of a square, as for example D.G. accoumptyng those two lines to bée equal, the other side of the square is D.E. which is equall to the whole line A.B.

Now may it appeare to your eye, that the greate square made of the whole line A.B, & of one of his parts that is A.C

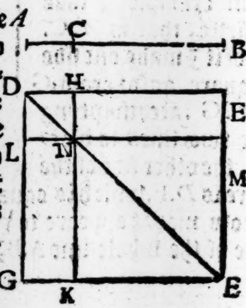
(which is equall with D.G.) is equall to two partiall squares, whereof the one is made of the saied greater portion A. C. in as much as not onely D. G beyng one of his sides, but also D. H. beyng the other side, are eche of them equall to A. C. The seconde square is H. E. F. K, in which the one side H. E. is equall to C. B. beyng the lesser parte of the line A. B. and E. F. is equall to A. C. which is the greater parte of the same line. So that those two squares D. H. K. G. and H. E. F. K. be both of them no more then the greater square D. E. F. G. accordyng to the wordes of the Theoreme afore saied.

The. xxxviii. Theoreme.

If a righte line bee deuided by chaunce, into partes, the square that is made of that whole line, is equall to bothe the squares that are made of eche parte of the line, and more ouer to twoo squares made of the one portion of the deuided line ioyned with the other in square.

¶Example.

Lette the deuided line be A.B. and parted in C. into two partes: Now saieth the Theoreme, that the square of the whole line A.B. is as muche iuste as the square of A. C. and the square of C. B, eche by it selfe, and more ouer by as much twise, as A.C. and C.B. ioyned in one square will make



For

Note this Theoreme

For as you see, the great square D.E.F.G, conteine tin hym fower lesser squares, of which the firste and the greatest is N.M.E.K and is equall to the square of the line A.C. The second square is the least of them all, that is D.H.L.N, and it is equall to the square of the line C.B. Then are there two other long squares both of one bignesse, that is H.E.N.M. and L.N.G.K. eche of them both hauyng two sides equall to A.C the longer parte of the deuided line, and there other two sides equall to C.B, beyng the shorter part of the saied line A.B.

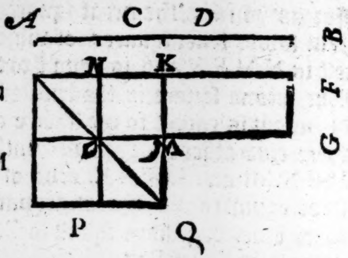
So is that greatest square, beyng made of the whole line A.B, equall to the two squares of eche of his partes seuerally, and more by as much iuste as two longe squares, made of the longer portion of the deuided line ioyned in square with the shorter parte of the same deuided line as the Theoreme would. And as here I haue putte an example of a line deuided into two partes so the Theoreme is true of all deuided lines, of what number so euer the partes bee, fower fiue, or sixe &c.

This Theoreme hath greate vse, not onely in Geometrie but allso in Arithmetike.

The.xxxix. Theoreme.

If a right line bee deuided into twoo equall partes, and one of these twoo partes deuided againe into twoo other partes, as happeneth, the long square that is made of the third, or later parte of that deuided line, with the residue of the same line, and the square of the midle moste parte, are both together equall to the square of halfe the first line.

Example.



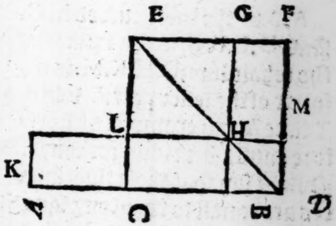
The line *A*. *B*. is deuided into two equall partes in C, and that parte C.B. is deuided againe as happeneth in *D*. Wherefore saieth the *T*heoreme, that the longe square made of D.*B*. and *A*. D, with the square of C.D. (which is the middle portion) shall both bée equall to the square of halfe the line *A*. B. that is to say, to the square of *A*.C or els of C. *D*. which make all one. The longe square F.G.N.O. which is the long square that the *T*heoreme. speaketh of, is made of two long squares, whereof the first is F.G.M.*K*, and the seconde is *K*.N.O.M. The square of the middle portion is E.*K*. Q.L. Now by the *T*heoreme, that long square F. *G*.M.O. with the iust square L. M.O. P, muste bée equall to the greate square E.*K*.Q.L. which thyng because it séemeth somewhat difficult to vnderstande, although I intende not here to make demonstration of the *T*heoremes, because it is appoincted to bée doen in the newe edition of Euclide, yet I will shewe you briefly how the equalitie of the partes doeth stand. And first I say, that where the comparison of equalitie is made, betwene the greate square (which is made of halfe the line A.B.) and two other, whereof the first is the long square F.*G*.N.O. and the second is the full square L M.O.P, which is one portion of the greate square all readie, and so is that longe square *K*.N.M O beyng a parcell also of the longe square. F.*G*.N.O. Where fore as those two partes are common to bothe partes compared in equalitie, and therefore béyng bothe abated from eche parte, if the reste of bothe eche other partes bée equall, then were those whole partes equall before: Nowe the reste of the greate square, those two lesser

ser squares beyng taken away. is that long square E.N.P.Q which is equall to the long square F.G.K.M being the rest of the other parte. And that they two bee equall, their sides doe declare. For the longest lines that is F K. and E.Q are equall and so are the shorter lines. F, G, and E.N. and so appeareth the truth of the Theoreme.

The. xl. Theoreme.

If a right line bee deuided into twoo euen partes, and an other right line annexed to one ende of that line, so that it make on right line with the firste. The long square that is made of this whole line so augmented, and the portion that is added, with the square of halfe the right line, shall bee equall to the square of that line, which is compounded of halfe the firste line, and the parte newly added.

Example.



The firste line proponed is A.B. and it is deuided into twoo equall partes in C, and an other right line, I meane *B.D.* annexed to one ende of the firste line.

Nowe say I, that the long square A. D. M. K. is made of the whole line so augmented, that is A. D, and the portion annexed, that is *D*.M, for *D*.M. is equall to B.*D* wherefore that long square A.D. M, K, with the square of halfe that first line, that is E.G.H.L. is equall to the greate square E, F, *D*, C. which square is made of y line C.D. that is

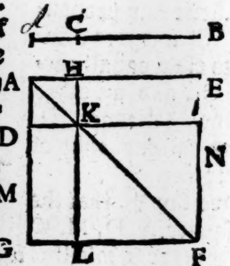
to

to say, of a line compounded of halfe the firste line, beyng C.B. and the portion annexed, that is B.D. And it is easely perceiued, if you consider that the longest square A.C.L.K. (which onely is lefte out of the greate square) hath an other long square equall to hym and so supplie his stede in the greate square, and that is G.F.M.H For their sides bee of like lines in length.

The.xlj. Theoreme.

If a right line bee deuided by chaunce, the square of the same whole line, and the square of one of his partes, are iuste equall to the long square of the whole line, and the saied parte twise taken, and more ouer to the square of the other parte of the saied line.

Example.



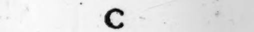
A.B. is the line deuided in C. And D.E.F.G is the square of the whole line, D.H.K.M. is the squar of the lesser portiō (which I take for an example) & therefore muste bee twise reckened. Now I say that those twoo squares are equall to twoo long squares of the whole line A.B. & his saied portion A C and also to the square of ỹ other portion of the saied first line, which portion is C.B. and his square K.N.F.L. In this Theoreme there is no difficultie, if you consider that the little square D.H.K.M. is

is fower tymes reckened, that is to saye, first of all, as a part of the greatest square, which is D.E.F.G. Secondly, he is reckened by hym self. Thirdly, he is accompted as parcell of the longe square D.E.N.M. And fourthly, he is taken as a parte of the other long square D.H.L.G, so that in as muche as he is twise reckened in one parte of the comparison of equalitie, and twise also in the seconde parte, there can rise none occasion of errour, or doubtfulnesse thereby.

¶The xlii. Theoreme.

If a right line bee deuided as chaunce happeneth the fower long squares, that maye bee made of that whole line and one of his partes, with the square of the other parte, shall bee equall to the square that is made of the whole line, and the saied firste portion ioyned to hym in length, as one whole line.

Example



The firste line is A.B. and is deuided by C. into twoo vnequall partes as happeneth, the long square of it, and his lesser portion A.C, is fower tymes drawen, the firste is E.G.M.K. the seconde is K.M.Q.O. the thirde is H.K.R.S. and the fowerth is K.L.S.T. And where as it appeareth that one of the little squares (I meane K.L.P.O.) is reckened twise,

 once

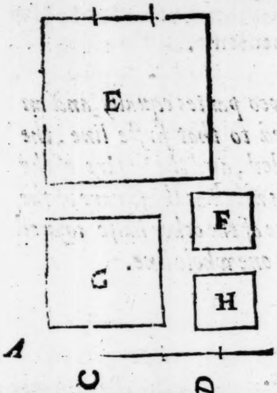
once as parcell of the seconde long square, and againe as part of the thirde long square, to auoide ambiguitie, you maye place one in steede of it, an other square of equalitie with it, that is to saye, D.E.K.H, which was at no tyme accompting as parcell of any of theim, and then haue you fower longe squares distinctly made of the whole line A.B, and his lesser portion A.C. And within them is there a greate full quare P. Q. T. V. which is the iuste square of B. C. beyng the greater portion of the line A. B. And that those fiue squares, doe make iuste as muche as the whole square of that longer line D.G (which is as long as A.B, and A.C. ioyned together) it maye bee iudged easily by the eye, sithe that one greate square doeth comprehende in it all the other fiue squares, that is to saye, fower long squares (as is before mentioned) and one full square, which is the intente of the Theoreme.

¶The.xliij. Theoreme.

If a right line bee parted into twoo equall partes firste, and one of those partes againe into other twoo partes, as chaunce happeneth, the square that is made of the laste parte of the line so deuided, and the square of the residue of that whole line, are double the square of halfe that line and to the square of the middle portion of the same line.

Example.

The line to bee deuided is A. B, and is parted in C. into twoo equall partes, and then C. B, is deuided againe into twoo partes in D, so the meanyng of the Theoreme, is that the square of D.B, which is the latter parte of the line, and the



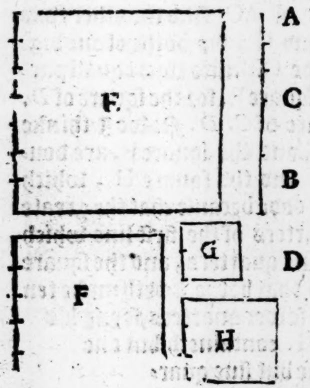
the square of A.D, whiche is the residue of the whole line. Those two squares, I say, are doubl to the square of the one halfe of the line, and to the squar of C.D, which is ẏ midl portiō of these thrée deuisions. Which thing ẏ you may more easilie perceiue, I haue drawen fower Square wherof the greatest béyng marked with E. is ẏ square of A.D. The nexte, which is marked with G, is the square of halfe the line, that is, of A.C. And the other two little squares marked with F. and H, bée bothe of one bignesse, by reason that I did deuide C.B. into two equall partes, so that you maye take the square F. for the square of D. B, and the square H. for the square of C. D. Nowe I thinke you doubte not, but the square E. and the square F, are double so much as the square G. and the square H, which thing the easier is to be vnderstoode because that the greate square hath in his side thrée quarters of the first line, which multiplied by it self, maketh nine quarters, and the square F. containeth but one quarter so that bothe dooeth make ten quarters. Then G. containeth fower quarters, seyng his side containeth two, and H. containeth but one quarter, which bothe make but fiue quarters, and that is but halfe of tenne.

Whereby you maye easily coniecture, that the meaning of the Theoreme is verified in the figures of this example.

The xliiii. Theoreme.

If a right line bee deuided into twoo partes equally, and an other portion of a right line annexed to that firſte line, the ſquare of this whole line ſo compounded, and the ſquare of the portion that is annexed, are double as much as the ſquare of the halfe of the firſte line, and the ſquare of the other halfe ioyned in one with the annexed portion, as one whole line.

Example.



The line is A. B and is deuided firſte into two equall partes in C. and then is there annexed to it an other portion, which is B. D. Now ſaith the Theoreme, that the ſquare of A.D. and the ſquare of B. D, are double to the ſquare of A. C. and to the ſquare of C. D. The line. A . B . containing foure partes, then muſt needes his halfe containe two partes, of ſuche partes I ſuppoſe B.D, (which is the annexed line) to containe three, ſo ſhall the hole line comprehend ſeuen partes, and his ſquare fortie and nine partes, wherevnto if you adde the ſquare of the annexed line, which maketh nine, then thoſe both

bothe dwe yeelde fiftie and eight, which must bée double to the square of the halfe line with the anexed portion. The halfe line by it self containeth but two partes, and therefore his square doeth make fower. The halfe line with the anexed portion containeth fiue, and the square of it is fiue and twentie, now putte fower to fiue and twentye, and it maketh iuste twentye and nine, the euen halfe of fiftie and eight, whereby appeareth the truthe of the *Theoreme.*

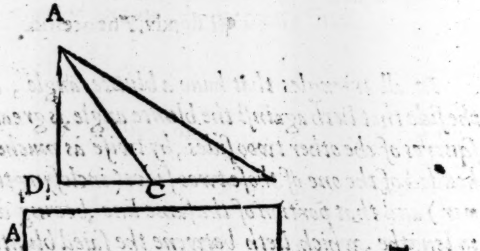
¶*The*.xlv.*Theoreme.*

In all triangles that haue a blunte angle, the square of the side that lieth against the blunte angle, is greater then twoo squares of the other twoo sides, by twise as muche as is comprehended of the one of those twoo sides (inclosyng the blunte corner) and that portion of the same line, beeyng drawen foorth in lengthe, which lieth betwene the saied blunte corner, and a perpendiculare line lightyng on it, and drawen from one of the sharpe angles of the foresaied triangle.

¶Example.

For the decla ration of this *Theoreme*, and the nexte also, whose vse are wonderfull in the practise of Geometrie, and in measuryng especially, it shall bée néedefull to declare that euery triangle that hath no right angle, as those bée which are called (as in the boke of practise is declared) sharpe cornered triangles, and blunte cornered triangles, yet maye they bée brought to haue a right angle, either by partyng them into twoo lesser triangles, or els by addyng

an other triangle vnto them, which maye be a greate helpe for the aide of measuryng, as more largelie shall bée sette forthe in the bóke of measuryng. But for this presente place, this fourme will I vse, which *Theon* also vseth) to adde one triangle vnto an other to bryng the blunte corne-red triangle vnto a right angled triangle, whereby the pro-portion of the squares of the sides in such a blunte cornered triangle, maye the better bée knowen.



First there fore I sette forth the tri-angle A.B.C. whose corner by C. is a blūt corner, as you maye well iudge, then to make an o-ther triangle, of it with a right angle, I muste drawe forth the side B.C. vnto D. and from the sharpe corner by A. I bring a plumbe line or perpendi-culare on D. And so is ther now a newe triangle A.B D whose angle by D. is a right angle. Now according to the meanyng of the *Theoreme*, I saye, that in the firste triangle A.B,C, because it hath a blunte corner at C, the square of the

the

the line A. B. which lieth againste the sayed blunte corner, is more then the square of the line A. C, and also of the line B. C. (which inclose the blunt corner) by as much as will amounte twise of the line B. C, and that portion D. C, which lieth betwene the blunte angle by C, and the perpendicular line A. D.

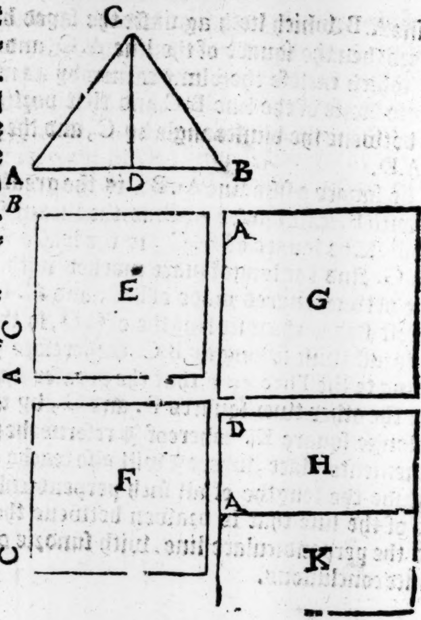
The square of the line A. B, is the greate square marked with E. The square A. C. is the meane square marked with F. The square of B. C, is the leaste square marked with G. And the longe square marked with K, is sette in steede of twoo squares made of B. C, and C. D. For as the shortest side is the iuste lengthe of C. D, so the other longer side is iust twise so long as B. C. Wherefore I saye now according to the Theoreme, that the greater square E, is more then the other twoo squares F. and G, by the quantitie of the longe square K. whereof I reserue the proofe to a more conueniente place, where I will also teache the reason how to finde the lengthe of all such perpendiculare lines, and also of the line that is drawen betwene the blunte angle, and the perpendiculare line, with sundrie other verie pleasaunte conclusions.

The. xlvi. Theoreme.

In sharpe cornered triangles, the square of any side that lieth againste a sharpe corner, is lesser then twoo squares of the other twoo sides, by as muche as is comprised twise in the long square of that side, on which the perpendiculare line falleth, and the portion of that same line, lying betwene the perpendicular, and the foresaied sharpe corner.

Example.

First



First I set forthe the triangle A.B.C.and in it I drawe a plūbe line from the angle C.vnto the line A.B. and it lighteth in D. Nowe by the Theoreme, the square of B.C.is not so muche as the square of the other two sides, that of B.A.and of A.C.by as much as it is twise contained in the lōgsquare made of A.B. and A.D.A.B.beyng the line or side, on which the perpendicular line falleth, and A.D.beyng that portion of thesame line, which doeth lye betwene the perpendicular line, and the sated sharpe angle limitted, which angle is by A.

For declaration of the figures, the square marked with E.is the square of B.C, which is the side that lieth againste the sharpe angle, the square marked with G.is the square of A.B and the square marked with F.is the square of A.C., the whole line A.B, and one of his portions A.D. And truthe it is that the square E, is lesser then the other two squares C.and F.by the quantitie of those two long squares H.and K

Whereby

Whereby you may consider againe, an other proportion of equalitie, that is to say, that the square E. with the two longe squares H.K. are iust equall to the other two squares G. and F. And so may you make, as it were an other Theoreme. That in all sharpe cornered triangles, where a perpendicular line is drawen from one angle to the side that lieth against it, the square of anie one side, with the two longe squares made of that whole line, where one the perpendicular line doth light, and of that portion of it, which ioyneth to that side, whose square is alreadie taken, those three figures, I saie are equall to the two squares, of the other two sides of the triangle. In which you must understande, that the side on which the perpendicular falleth, is thrire used, yet is his square but once mecioned. for twise he is taken for one side of the two long squares. And as I haue thus made as it were an other Theoreme out of this fortie and sixe Theoreme, so might I out of it, and the other that goeth nexte before, make as many as would suffice for a whole booke, so that when they shall be applyed to practise and consequently to expresse theire benefite, no man that hath not well weighed their wounderfull commoditie, would creadite the possibilitie of their wounderfull use, and large aide in knowledge. But all this will I committe to a place conueniente.

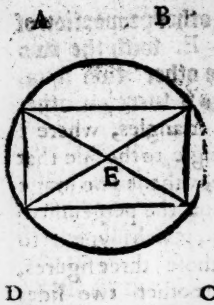
The.xlvii.Theoreme.

If two pointes be marked in the circumference of a circle, and a right line drawen from the one the other that line must needes fall within the circle.

Example.

The circle is A.B.C.D. the two pointes are A,B, the

 right



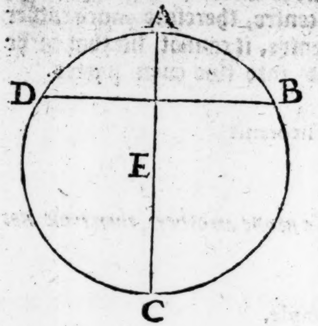
right line that is drawen from the one to the other, is the line A.B. which as you see, must needes light within the circle So if you putte the pointes to bee A.D. or D.C, or A.C., either B.C, or B.D, in any of these cases you see, that the line that is drawen from the one pricke, to the other, doth euermore runne within the edge of the circle, els can be no right line. How be it that a cooked line especially being more crooked then the portion of the circumference, may be drawen from pointe to pointe, with out the circle But the Theoreme speaketh onely of right lines, and not of crooked lines.

The xlviii Theoreme.

If a right line passing the centre of a circle, doe crosse another right line within the same circle, passing beside the centre if he deuide the said line into two equall partes. then doe they make all their angles right, And contrary waies, if they make all there angles right, then doth the longer line, cut the shorter in two partes.

¶Example.

The circle is A.B.C.D, the line that passeth by the centre is A.E.C, the line that goeth beside the centre is D. B. Now saie I, that line A,E.C.. doeth cutte that other line

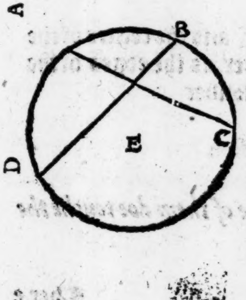


line D.B, into two iust partes. and ther fore all their fower angles are right an gles. And contrarie waies, because all their ãgles ar eright angles, therefore it must bee true, that the greater cut teth the lesser into two equall partes, accor ding as the Theore me would

The xlix. Theoreme

If two right light lines drawen in a circle, doe crosse one an o-ther, and doe not passe by the centre, euery of them doeth not deuide the other in ii equall portions.

Example.



The circle is A.B.C.D. and the centre is E. the one line A.C. and the other B. D. which two li-nes crosse one an other, but yet they goe not by the centre, wher fore according to the wordes of the Theoreme, eche of them doth cutte the other into equall porti-ons. For as you may easily iudge A.C. hath one portion longer and an other shorter, and so likewise B, D. Howbeit, it is not so to be vnderstãd, but one of them may be deuided into ij euen parts,

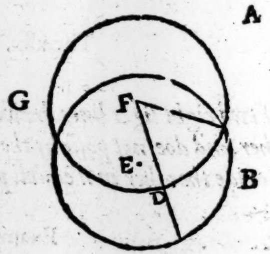
but both to bée cutte equally in the middle, is not possible, vnlesse both passe through the centre, therefore much rather when both besides the centre, it cannot bée that eache of them should be iustly parted into two euen partes.

The .l.Theoreme

If two circles crosse and c utone an other , then haue not they both one centre.

Example,

This Theoreme séemeth of it self so manifest, that it nedeth neither demonstration. neither declaration. Yet for the plaine vnderstanding of it, I haue set forth a figure here, where two circeles bée drawen , so that one of them doeth crosse the other (as you sée) in the pointes *B*. and G, and thire centres appeare at the first sighte to bée diuers. For the centre of the one is F· and the centre of the other is E.which differ as farre a sonder,as the edges of the circles, where they bée most distant in sonder

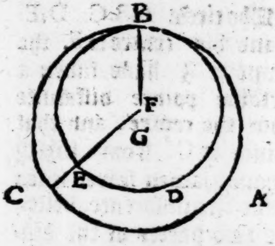


The li.conclusion

If two circles bee so drawen,that one of them doe touche the other , then haue not they one centre.

Example.

There

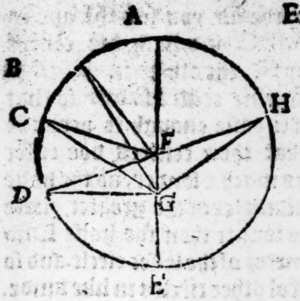


There are two Circles made as you see, the one is A.B.C. and hath his centre by G. the other in B, D.E an dhis centre is by F. so that it is easie enough to perceiue that their centres doe differ as much a sonder, as the halfe diameter of the greater circle is longer then the halfe Diameter of the lesser circle. and so must it nedes be thoght & saied of al other circles in like kinde.

¶The. liij. Theoreme.

If any certaine point be assigned in the diameter of a circle distaunte from the centre of the said circle, and from that point diuers lines drawen to the edge and circumference of the same circle, the longest line is that which passeth by the centre, and the shortest is the residewe of the same line. And of all the other lines that is euer the greatest, that is nighest to the line, which passeth by the centre. And cōtrarie waise, that is shortest that is farthest from it, And amongest them all there can be but only two equall together, and they must needes be so placed, that the shortest line shal be in the iust middle betwixte thē

 Exam-



Example.

The circle A.B.C. D.E. H.and his centre is F. the diameter I haue taken a certaine pointe distaunte from the centre, and that pointe is G. from which I haue drawen fower lines to the circumference, beside the two partes of the diameter, which maketh vp-sixte lines in all. Now for the diuersitie in quantitie of these lines, I say according to the Theoreme, that the line which goeth by the centre is the longest line, that is to say A.G.and the residewe of the same diametre being G.E, is the shortest line. And of all the other, that line is longeste, that is neerest vnto that parte of the diametre, which goeth by the centre, and that is shortest, that is farthest distaunte from it, wherefore I say, that G.B. is longer then G.C. and therefore much more longer then G.D.. sith G. C. also is longer then G.D. and by this may you sone perceiue, that it is not possible to drawe two lines, on any one side of the diameter, which might be equall in lengthe together, but on the one side of the diameter, may you easilie make one line equall to an other, on the other side of the same diametre, as you see in this example. G.H. to bee equall to G.C. betweene which the line, G.E. (as the shortest in all the circle) doeth stande euen distaunte from eche of them, and that is the precise knowledge of their equalitie if thy bee equallie distaunte from one halfe of the diameter. Where as contrarie waies, if the one bee neerer to any one halfe of the diameter then the other is, it is not possible that they two may bee equall in length, namely if they doe ende both in

the

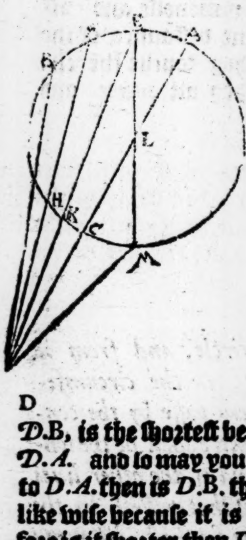
the circumference of the circle, and bee both drawen from one pointe in diametre, so that the saied pointe bee (as the Theoreme doeth suppose) somewhat distaunte from the centre of the saied circle, For if they bée drawen from the centre, then must they of necessitie bée all equall, howe many so euer they bée, as the definition of a circle doeth importe, without any regarde how nere so euer they bee to the Diametre, or haue distaunte from it. And here is to be noted, that in this Theoreme, by néerenesse and distaunce is understand, the néerenesse and distaunce of the extreame partes of those lines, where they touche the circumference. For at the other ende they all méete and touch.

Theliii, Theoreme,

If a pointe be marked without a circle, and from it diuers lines drawen crosse the circle, to the circumference on the other side, so that one of them passe by the centre, then that line which passeth by the centre, shall be the longest of all them that crosse the circle. And of the other lines those are longest, that be nexte vnto it that passeth by the centre. And those are shortest, that be farthest distaunte from it. But emonge those partes of those lines, which ende in the outward circumference, that is most shortest which is parte of the line that passeth by the centre, and amongst the other eche of them the neerer they are vnto it, the shorter

ter they, and the fathers from it, the longer they be. And emongest them al, there can not be more then two of any one in length, and they two must be on the two contrarie sides of the shortest line.

Example.



Take the circle to bée A B. C. and the pointe assigned without it bée D. Now say I that if there bée drawen sundrie lines from D. and crosse the circle, ending in the circumference on the contrarie side as here you sée, D. A. D. E. D. F. and D. B. then of all these lines the longest must néedes bée D. A. which goeth by the centre of the circle, and the nexte unto it that is D. E, is the longest amongst the reste. And contrary waies, D. B, is the shortest because it is the farthest distaunt from D. A. and so may you iudge of D. F. be cause it is nearer unto D. A. then is D. B. therefore it is longer then D. B. And like wise because it is farther from D. A. then D. E therefore is it shorter then D. E. Nowe for those partes of the lines, which be without the circle (as you sée) D. C, is the shortest, because it is the parte of that line, which passeth by the centre. And D. K. is nexte to it in distaunce, and therfore also in shortnesse, so D. G. is farthest from it in distance and therefore is the longest of them. Now D. H. being nerer then D, G is also shorter then it, and béeing farther of then

then D.K, is longer then it. So that for this parte of the Theoreme (as I thinke) you doe plainlie perceiue the truth thereof, so the residewe hath no difficultie. For seyng that the neerer any line is to D.C, (which ioyneth with the diameter) the shorter it is, and the farther of from it, the longer it is. And seyng two lines can not bée of like distaunce, beyng both on one side, therefore if they shall be of one length, and consequentlie of one distaunce, they muste néedes bée on contrarye sides of the saied line *D*.C. And so appeareth the meanyng of the whole *T*heoreme.

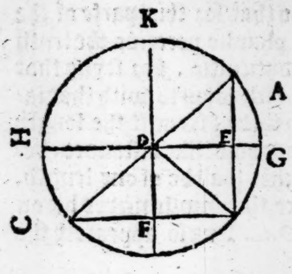
And of this *T*heoreme doeth there followe an other like, which you maye call, either a *T*heoreme by it self, or els a Corollarie vnto this laste *T*heoreme, I passe not so much for the name. But his sentence is this: when so euer any lines bee drawen from any poincte, without a circle, whether they crosse the circle, or ende in the vtter edge of his circumference, those twoo lines that bee equally distaunt from the leaste line are equall together, and contrary waies, if they bee equall together, they are also equally distaunt from that least line.

For the declaration of this proposition, it shall not néede to vse any other example, then that which is brought for the explication of this laste *T*heoreme, by which you may without any teachyng easily perceiue, bothe the meanyng, and also the truth of this proposition.

¶ *The.liiii.Theoreme.*

If a poincte bee sette in a circle, and from that poincte vnto the circumference many lines drawen, of which more then twoo are equall together, then is that poincte the centre of that circle.

¶Example.



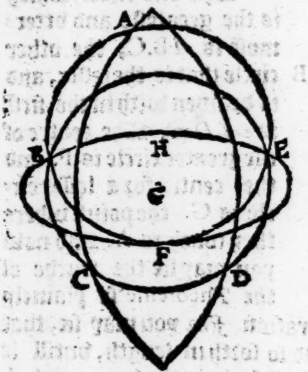
The circle is A B.C, and within it I haue sette forth for an example thrée prickes, which are D.E, and F, and from euery one of thē I haue drawen (at the leaste) fower lines vnto the circumference of the circle but from D, I haue drawen more, yet may it appeare readily vnto your eye, that of all the lines which bée drawen from E and F, vnto the circumference, there are but two equall and more can not bée, for G.F, nor E.H, hath none other equall to them, nor can not haue any, being drawen from the same pointe E. No more can L.E, or F.K. haue any line equall to either of them being drawen from the same point F. And yet from either of these two pointes, are their drawen two lines equall together, as A.E, is equall to E. B and B.F, is equall to F.C. but there can no thirde line bée drawen equall to either of these two couples, and that is, by reason that they be drawen from a pointe distaunt from the centre of the circle. But from D, although there bée seuen lines drawen to the circumference, yet all bee equall, because it is the centre of the circle. And therefore if you drawe neuer so many more from it vnto the circumference, al shal bée equall, so that this is the priuiledge (as it were of the centre) and therefore no other pointe can haue a boue two equall lines drawen from it vnto the circumference. And from all pointes you may drawe two equall lines to the circumference of the circle, whether that pointe bée within the circle, or without it.

The lv Theoreme

No circle can cutte an other circle, in more pointes then

then two.

Example

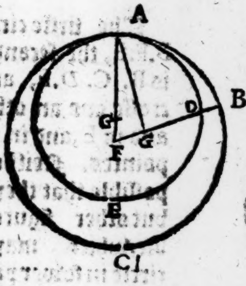


The firste circle is A. B.F.E, the seconde circle is B. C. D.E, and they crosse one an other in B and in E, and in no more pointes. Neither is it possible that they should, but other figures there bee, which may cutte a circle in fower partes, as you see in this example. Where I haue set forth one tunne fourme, and one eye fourme, and eche of them cutteth euery of their two circles into foure partes. But as they be irregulare formes, that is to say, such formes as haue no precise measure neither proportion in their draughte, so can there scarsely be made any certaine Theoreme, of them. But circles are regulare fourmes, that is to say, such formes as haue in their protacture, a iust and certaine proportion, so that certaine & determinate truthes may be affirmed of them, sithe they are uniforme and unchangeable.

The.lvi.Theoreme.

If two circles be so drawen, that the one be within the other and that they touch one an other: if a line be drawen by both their centres, and so forth in length, that line shall rune to that pointe, where the circles doe touch.

¶Example.



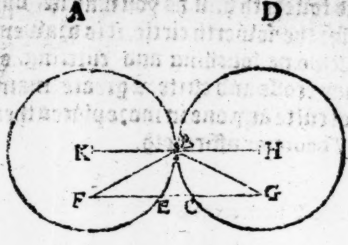
The one circle, which is the greateste and vttermost is A.B.C, the other circle that is the lesser, and is drawen with in the first is A.D.E, The centre of the greater circle is F. and the centre for a lesser circle is G. the point where they touch is A. And now you may see the truthe of the Theoreme so plainely that it needeth no farther declaration. For you may see, that drawing a line from F. to G, and so forth in length, vntill it come to the circumference, it will light in the very point A, where the circles touch one an other.

The.lxvj.Theoreme.

If two circles be drawen so one without an other, that their edges doe touch, and right line be drawen from the centre of the one to the centre of the other, that line shall passe by the place of their touching.

Example.

The first circle is A.B.E, and his centre is K, The seconde circle is D.B.C. and his centre is H. the pointe where they doe touch is B. Now doe you see that the line K,H, which

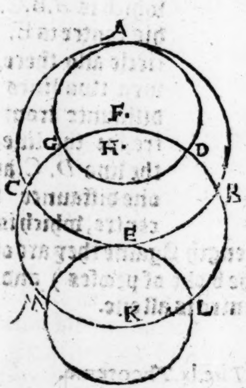


which is drawen from K, that is centre of the first circle, unto H, being centre of the second circle, doeth passe (as it must nedes by the pointe B.) which is the verie pointe where they doe touche together.

The.lviii.Theoreme.

One circle cannot touch an other in more pointes then one, whether they touch within, or without.

Example.



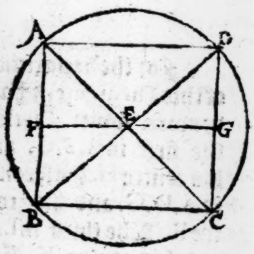
For the declaration of this Theoreme, I haue drawen fower Circles, the first is A.*B*.C, and his centre H. the seconde is A.D.G. and his centre F. The third is L.M and his centre K. The fowerth is *D*. G.L.M, and his centre E, Now as you perceiue the second circle A. D. G, touching the first in the inner side, in so much as as it is drawen with in the other and yet it toucheth but in one pointe, that is to say in *A*. so likewise the thirde circle L.M, is drawen with-

without the first circle, and toucheth him as you may see, but in one place. And now as for the fowerth circle, it is drawen, to declare the diuersitie betwene touching and cuttting, or crossing. For one circle may crosse and cutte a greate many other circles, yet can he not cutte any one in more places then two, as the fiue and fiftie Theoreme affirmeth.

The.lix. Theoreme.

In euery circle those lines are to be counted equall, which are in like distance from the centre. And contrarie waies, they are in like distaunce from the centre which be equall.

Example.



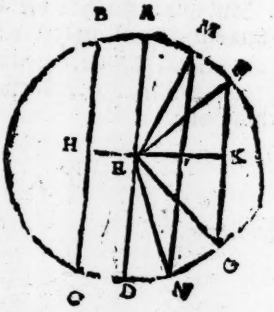
In this figure you see firste the circle drawen which is A.B.C.D. and his centre is E, In this circle also there are drawen two lines equally distaunte from the centre, for the line A.B, and the line D. C. are iuste of one distaunce from the centre, which is E, and therefore are they of one length. Againe they are of one length (as shall be proued in the boke of profes) and therefore their distance from their centre is all one.

The.lx. Theoreme.

In euerie circle the longest line is the diameter, and of all the other lines they are still longest that be nexte

vnto the centre, and they bee the shortest, that bee farthest distaunte from it.

Example.



In this circle A.B.C D. I haue drawen firste the diametre, which is A.D, which passeth (as it must) by the centre E. Then haue I drawen ij. other lines as M.N, which is nearer the centre, and F.G, that is farther from the centre. The fowerth line also on the other side of the diametre that is B.C. is nearer to the centre then the line F.G, for it is like distance as the line M.N, Now say I that A.D, being the diameter, is the longest of all those lines, and also of any other that may be drawen within that circle. And the other line M.N, is longer then F.G, because it is nearer to the centre of the circle then F. G. Also the line F.G. is shorter then the line B.C, for because it is farther from the centre then is the line B.C, And thus may you iudge of all lines drawen in any circle, how to knowe the proportion of their length, by the proportion of their distance, and contrary waies, how to discerne the proportion of their distance by their lengthes, if you knowe the proportion of their lengthe. And to speake of it by the way, it is a marueilous thing to consider, that a man may knowe an exacte proportion betweene two thinges, and yet cannot name nor attaine the precise quantitie of those two thinges. As for example, If two squares bee sette forthe, whereof the one containeth in it v. square feete, and the other containeth fiue and fourtie fote, of like square feete, I am no[t]

not able to tell, no nor yet any man liuing, what is the precise measure of the sides, of any of those two squares, and yet I can proue by vnfallible reason, that their sides bee in a triple proportion, that is to say, that the side of the greatest square (which containeth fowertie and fiue foote) is three times so long iuste, as the side of the lesser square that concludeth but fiue foote. But this seemeth to bee spoken out of season in this place therefore I will omit it now, referuing the exacter declaration thereof, to a more conuenient place and time, and will procede with the residewe of the Theoremes appointed for this booke.

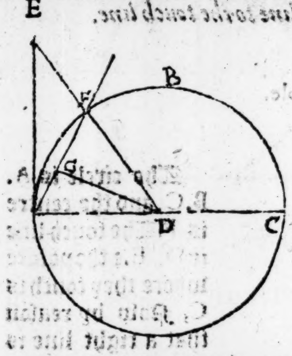
The.lxi.Theoreme.

If a right line bee drawen at any ende of a diametre in perpendicular fourme, and doe make a right angle with the diametre, that right line shall light without the circle, and yet so ioyntly knitte to it, that it is not possible to drawe any other right line betweene that said line, and the circumference of the circle. And the angle that is made in the semicircle is greater the any sharpe angle, that may be made of right lines, but the other angle without, is lesser the͂ any can be made of right lines.

Example.

In this circle A.B.C, the diameter is A.C, the perpendiculare line, which maketh a right angle with the diameter is E.A, which line falleth without the circle, and yet ioyneth so exactly vnto it, that it is not possible to drawe an other

ther right line, betwen the circumference of the circle and it, which thyng is so plainly séene of the eye, that it nedeth no farther declaration. For euery manne will easily consent, that betwen the croked line A.F, (which is a parte of the circumferenc of the circle) and A.E, (which is the saied perpendiculare line) there can none other line bée drawen in that place, where they make the angle. Now for the residue of the Theoreme The angle D.A.B. which is made in the semicircle, is greater then any sharpe angle, that may bée made of righte lines, and yet it is a sharpe angle also, in as much as it is lesser then a right angle, which is the angle E.A.D, and the residue of that right angle, which lieth without the circle, that is to say, E.A.B, is lesser then any sharpe angle that can bée made of right lines also. For as it was before rehearsed, there can no right line bée drawen to the angle, betwene the circumference and the right line E.A. Then muste it néedes followe, that there can bée made no lesser angle of right lines. And againe, if there can be no lesser then the one, then doeth it sone appere, that there can be no greater then the other, for they twoo doe make the whole right angle, so that if any corner could be made greater then the other parte, then should the residue bée lesser then the other parte, so that either both partes muste bée false, or els both graunted to bée true.

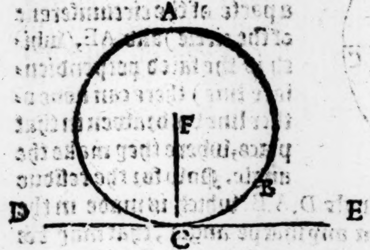


The.lxii. Theoreme.

If a right line dooe touche a circle, and an other righte line drawen from the centre of the circle, to the

pointe where they touche that line which is drawen from the centre, shall be a perpendiculare line to the touch line.

Example.



The circle is A.B.C. and the centre is F. The touch line is D. E. & the pointe where they touch is C, Now by reason that a right line is drawẽ from the centre F. unto C, which is the poincte of the touch therfore saieth ẏ *Theoreme*, that the saied line F.C. must nedes bee a perpendicalare line unto the touch line D.E.

The.lxiii.*Theoreme.*

A *If a right line doe touch a circle, and an other right line bee drawen from the poincte of their touchyng, so that it doe make right corners with the touche line, then shall the centre of the circle bee in that same line, so drawen.*

Example.

The circle is A.B.C, and the centre of it is G. The touch line is *D*.C.E. and the poincte where it toucheth, is C. Now it appeareth manifeste, that if a right bee drawen from the poincte

pointe where the touch line doeth ioyne with the circle, and that the saied line do make right corners with the touch line, then must it nedes goe by the centre of the circle, and then consequently it must haue the saied centre in him. For if the said line should goe beside the centre as F.C. doth, then doth it not make right angles with the touch line, which in the *Theoreme* is supposed.

The xxxii. *Theoreme*.

If an angle bee made on the centre of a circle, and an other angle made on the circumference of the same circle, and there ground line be one common portion of the circumference, then is the angle on the centre twise so greate as the other angle on the circumference.

Example.

The circle is A.B.C.*D*, and his centre is E, the angle on the centre is C.E.D and the angle one the circumference is C. A. D. their common ground line is C.F.D. Now saie I that the angle C. E.*D*. which is one the centre, is twise so greate as the angle C.A. *D*.which is on the circumference.

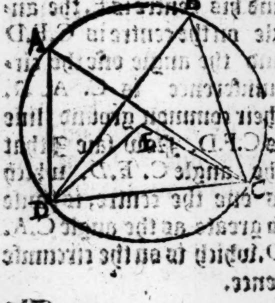
 The

The.lv.Theoreme.

Those angles, which bee made in one cantle of a circle, must needes bee equall together.

¶Example.

Before I declare this Theoreme by an example, it shall bee needefull to declare, what is to bee understoode by the wordes of this Theoreme: For the sentence can not bee knowen, unlesse the verie meanyng of the wordes be first understoode. Therefore when it speaketh of angles made in one cantle of a circle, it is this to bee understoode, that the angle must touch the circumference: and the lines that doe inclose that angle, must bee drawen to the extremities of that line, which maketh the cantle of the circle. So that if any angle doe not touch the circumference, or if the lines that inclose that angle, doe ende in the extremities of the corde line, but ende either in some other parte of the saied corde, or in the circumference, or that any of them doe so ende, then is not that angle accoumpted to bee drawen in the saied cantle of the circle. And this promised, nowe will I come to the meanyng of the Theoreme, I sette forth a circle, which is A.B.C.D, and his Centre E, in this circle I drawe a line C.D, whereby there are made two cantels, a more and a lesser. The lesser is D.E.C, and the greater is D.A.B.C. In this greater cantle I drawe two angles, the firste is D.A.C: and the seconde is D.B.C, which two angles by reason they are made bothe



in one cantle of a circle (that is the cantle D.A.B.C.) therefore are they bothe equall. Now doeth there appere an other triangle, whose angle lighteth on the centre of the circle and that triangle is D.E.C, whose angle is double to the other angles, as is declared in the sixtie and fower Theoreme, which maye stande well enoughe with this Theoreme, for it is not made in this cantle of the circle, as the other are, by reason that his angle doeth not light in the circumference of the circle, but on the centre it self.

The.lxvi. Theoreme.

Euerie figure of fower sides, drawen in a circle, hath his two contrarie angles, equall vnto twoo right angles.

Example.

The circle is A.B. C. D. & the figure of fower sides in it, is made of the sides B.C, & C.D, and D.A, & A.B. Now if you take any two angles ye be contrary, as ye angle by A, and the ãgle by C. I say that those two bee equall to two right angles. Also if you take ye angle by B. & the angle by D. which two are also contrary, those two angles are likewise equall to two right angles But if any man will take the angle by A, with the angle by B, or D. they can not be accoumpted contrarie, no more is not the angle by C, estemed contrary to the angle by B, or yet to the angle by D, for they onely bee accoumpted contrary angles. which haue no one line common to them both.

 Such

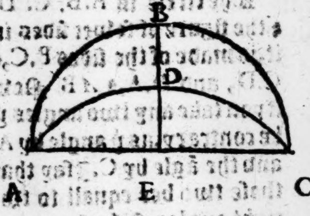
Such is the angle by *A*, in respecte of the angle by C, for their both lines bée distincte, whereas the angle by A, and the angle by *D*. haue one common line *A,D*. and therefore cannot be accompted contrarie angles. So the angle by D. and the angle by C haue D.C. as a common line, and therefore bée not contrarie angles. And this may you iudge of the residewe, by like reason.

The lxvii Theoreme.

Vpon one right line there cannot be made two cantles of circles, like and vnequall, and drawen toward on part.

Example.

Cantles of circles bee then called like, when the angles that are made in them be equall. But now for the Theoreme, lette the right line bee A.E.C, on which I drawe a cantle of a circle, which is *A*.B.C. Now saith the *Theo*reme that it is not possible to drawe an other cantle of a circle, which shall be vnequall vnto this first cantle, that is to say either greater or lesser then it, and yet be like it also, that is to say, that the angle in the one, shall be equall to the angle in the other. For as in this example you sée a lesser cantle drawen also, that is A.D.C, so if an angle were made in it, that angle would be greater then the angle made in the cantle A.*B*. C, and therfore can not they bee called like cantles, but and if any other cantle were made greater then the first, then would the angle be lesser, then that in the first



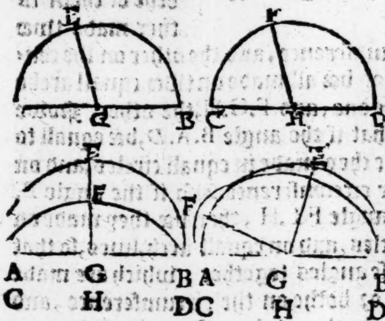
firſt, and ſo neither a leſſer neither a greater cantle can bée made vpon one line with an other, but it will be vnlike to it alſo.

¶The.lxviij. Theoreme.

Like cantles of circles made on equall right lines, are equall together.

Example.

What is meante by like cantles you haue heard before, and it is eaſie to vnderſtand, that ſuch figures are called equall, that bée of one bigneſſe, ſo that the one is neither greter, neither leſſer then the other. And in this kinde of compariſon, they muſt ſo agrée, that if the one bee laied on the other, they ſhall exactlie agrée, in all their boundes, ſo that neither ſhall exceade other.

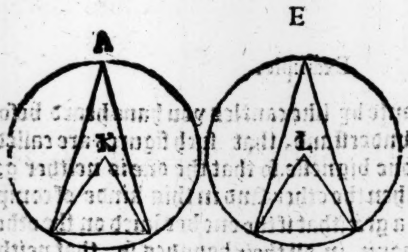


Now for the examples of the Theoreme, I haue ſette forth diuerſe varieties of cantles of circles, amongeſt which the firſt and ſeconde are made vpon equall lines, and are alſo both equall and like. The thirde couple are ioyned in one and be neither equall, neither like, but expreſſing an abſurde deformitie, which would follow if this Theoreme were not true. And ſo in the fowerth couple you may ſée, that becauſe they are not equall cantles, therefore can not they be like cantles for neceſſarily it goeth together, that all cantles of the circle made vpon equall right lines, if they be like, they muſt be equall alſo.

The.lxix.Theoreme.

In equall circles, such angles as bee equall are made vppon equall arch lines of the circumference, whether the angle lighte on the circumference, or on the centre.

Example.



First I haue set for an example two equal circles, that is A.B.C.D, whos centre is K, and the second circle E.F G.H, and his centre L, and in eche of them is ther made two angles, one on the circumference, and the other on the centre of ech circle, and they bee all made on two equall arche lines, that is B.C.D, the one, and F.G.H, the other. Nowe saieth the Theoreme, that if the angle B.A.D, bee equall to the angle F.E.H, then are they made in equall circles, and on equal arch lines of their circumference. Also if the angle B.K.D, bee equall to the angle F.L.H, then bee they made on the centres of equall circles, and on equall arch, lines, so that you must compare those angles together, which are made bothe on the centres, or bothe on the circumference, and may not conferre those angles, whereof one is drawen on the circumference, and the other on the centre. For euermore the angle on the centre in such sorte, shall bee double to the angle on the circumference, as is declared in the three score and fower Theoreme.

¶The.lxx.Theoreme.

In equall circles, those angles whiche bee made on equall arche lines, are euer equall together, whether they bee made on the centre, or on the circumference.

¶Example.

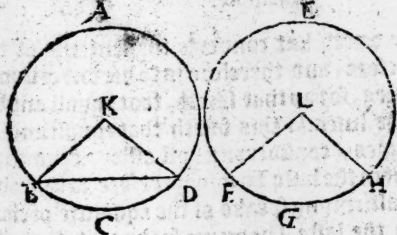
This Theoreme dooeth but conuerte the sentence of the laste Theoreme before, and therefore is to bee vnderstoode by the same examples, for as that saieth, that equall angles occupie equall arche lines: so this saieth, that equall angles cause equall angles, considerypng all other circumstances, as was taught in the laste Theoreme before, so that this Theoreme dooeth affirmyng speake of the equalitie of those angles, of which the laste Theoreme spake conditionally. And where the laste Theoreme spake affirmatiuely of the arche lines, this Theoreme spake conditionally of them, as thus: If the arche line B.C.D, be equall to the other arche line F.G.H, then is that angle B.A D, equall to the other angle F.E.H. Or els thus maye you declare it causally: Because the arch line B.C.D, is equall to the other arche line F.G.H, therefore is the angle B.K.D. equall to the angle F.L.H, considerypng that they are made on the centres of equall circles. And so of the other angles, because those two arche lines aforesaied are equall, therefore the angle D.A.B, is equall to the angle F.E.H, for as muche as they are made on those equall arche lines, and also on the circumference of equall circles. And thus these Theoremes doe one declare an other, and one verifie the other.

The lxxj. Theoreme.

In equall circles, equall right lines beyng drawen dooe cutte a waie equall arche lines from their circum-

ferences, so that the greater arche line of the one, is equall to the greater arche. line of the other, and the lesser to the lesser.

¶Example.



The circle A B.C.D. is made equall to the circle E.F.G.H and the righte line B.D. is equall to the right line F.H wherfore it followeth, that the two arche-lines of the circle A.B.D, which are cutte from his circumference by the right line B.B, are equall to two other arche lines of the circle E.F.H, beyng cutte from his circumferẽce. by the right line F.H, that is to saye, that the arche line B.A D, beyng the greater arche line of the firste circle is equall to the arche line F.E.H, beyng the greater arche line of the other circle. And so in like maner the lesser arche line of the firste circle, beyng B.C.B, is equall to the lesser arch line of the seconde circle, that is F.G.H.

¶The.lxxij.Theoreme.

In equall circles, vnder equall arche lines the right lines that bee drawen are equall together.

¶Example.

This Theoreme is none other, but the conuersion of the laste

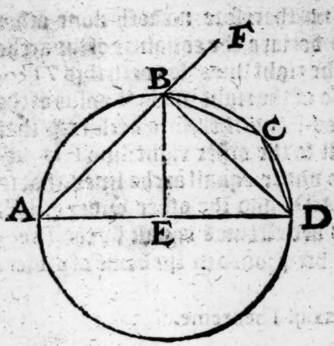
laste Theoreme before, and therefore nedeth none other example. For as that did declare the equalitie of the arche lines, by the equalnesse of the right lines, so doeth this Theoreme declare the equalnesse of the right lines, to ensue of the equalnesse of the arch lines, and therefore declareth that right line B.,D to bee equall to the other right line F.H, because they both are drawen vnder equall arche lines, that is to saye, the one vnder B.A.D, and the other vnder F.E.H, and those two arche lines are estemed equall by the Theoreme laste before, and shall bee proued in the boke of profes.

¶The.lxxiij. Theoreme.

In euery circle, the angle that is made in the halfe circle, is a iuste right angle, and the angle that is made in a cantle greater then the halfe circle, is lesser then a right angle, but that angle that is made in a cantle, lesser then the halfe circle, is greater then a right angle. And moreouer the angle of the greater cantle is greater then a right angle, and the angle of the lesser cantle, is lesser then a right angle.

¶Example.

In this proposition, it shall bee mete to note, that there is a greate diuersitie betwene an angle of a cātle, and an angle made in a cantle, and also betwene the angle of a semicircle, and the angle made in a semicircle. Also it is mete to note that all angles that be made in the parte of a circle, are made either in a semicircle (which is the iuste halfe circle) or els in a cantle of the circle, which cantle is either greater or lesser then the semicircle is, as in this figure annexed, you maye perceiue euery one of the thynges seuerally.



Firste the circle is, as you see, A.B.C.D, and his centre E, his diametre is A.D. Then is there a line drawen from A. to B and so forthe unto F. which is without the circle: and an other line also from B, to D, which maketh two cantles of the whole circle: The greater cantle is D.A.B and the lesser cãtle is B.C.D. In whiche lesse rcantle also there are two lines that make an angle, the one line is B.C, and the other line is C.D. Now to shewe the difference of the angle in a cantle, and an angle of a cantle: first for an exãple. I take the greater cantle B.A.D, in which is but one angle made, and that is the angle by A, which is made of the line A.B, and the line A.D. And this angle is therefore called an angle in a cantle. But nowe the same cantle hath two other angles, which be called the angles of that cantle, so the two angles made of the right line D.B, & the arch line D.A B, are the two angles of this cantle, whereof the one is by D, and the other is by B. Where you must remember that the angle by D, is made of the right line B.D, and the arche line D.A. And this angle is deuided by an other right line A.E. D, which in this case must bee omitted as no line. Also the angle by B, is made of the right line D. B and of the arche line B.A and although it bee deuided with two other right lines, of which the one is the right line B.A, and the other the right line B.E, yet in this case they are not to bee considered. And by this may you perceiue also, which be the angles of the lesser cantle, the first of them is made of the right line B.D, and of the arch line B.C, the secõd is made of the right line D.B, and of the arche line D.C. Then are there two o-

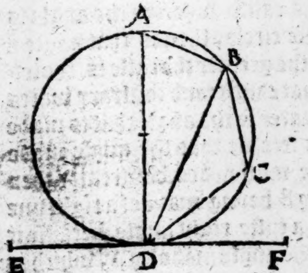
ther lines, whiche deuide those two corners, that is the line B.C, and the line C, D, which two lines doe meete in the poinct C, and there make an angle, which is called an angle made in that lesser cantle, but yet is not any angle of that cantle. And so haue you heard the difference betwene an angle in a cantle, and an angle of a cantle. And in like sort shall you iudge of the angle made in a semicircle, which is distinct from the angles of the semicircle. For in this figure, the angles of the semicircle are those angles, which bee by A, and D, and bee made of the right line A.D, beyng the diameter, and of the halfe circūference of the circle, but the angle made in the semicircle, is that angle by B, which is made of the right line A.B, and that other right line B.D, which as they meete in the circumference and make an angle, so they ende with their other extremities at the endes of the diameter. These things promised, now saye I touchyng the Theoreme that euery angle that is made in a semicircle is a righte angle, and if it bee made in any cantle of a circle, then muste it nedes be either a blunt angle, or els a sharpe angle and in no wise a righte angle. For if the cantle wherein the angle is made bee greater then the halfe circle, then is that angle a sharpe angle. And generally the greater the cātle is the lesser is the angle comprised in that cantle: and contrary waies the lesser any cantle is, the greater is the angle that is made in it. Wherefore it must nedes folowe, that the angle made in a cātle lesse then a semicircle, must nedes bee greater then a right angle. So the angle by B beyng made of a right line A.B and the right line B.D, is a iuste right angle, because it is made in a semicircle. But the angle made by A, which is made of the right line A.B, and of the right line A.D, is lesser then a right angle, and is named a sharp angle, for as much as it is made in a cantle of a circle, greater then a semicircle. And contrary wise the angle by C. beyng made of the right line B.C, and of the righte line C.D, is greater then a right angle, and is named a blunte angle, because it is made in a cantle of a circle, lesser then a semicircle. But now

touchyng the other angles of the cantles, I saye according to the Theoreme, that the two angles of the greater cantle which are by B. and D, as is before declared, are greater eche of them then a right angle. And the angles of the lesse, cantle, which are by the same letters B. and D, but bee on the other side of the corde are lesser eche of them then a right angle, and bee therefore sharpe corners.

¶The.lxxiiij.Theoreme.

If a righte line dooe touche a circle, and from the poincte where they touche, a right line be drawen crosse the circle, and deuide it, the angles that the saied line doeth make with the touche line, are equall to the angles, which are made in the cantles of the same circle, on the contrary sides of the line aforesaied.

Example.



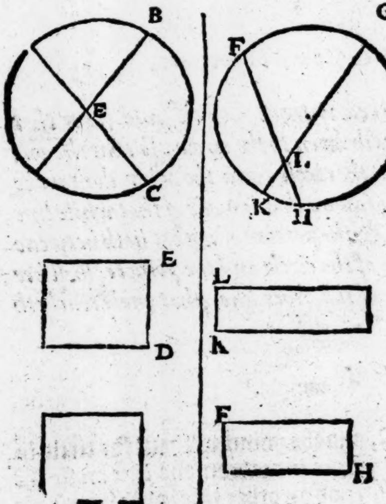
The circle is A.B.C.D and the touche line is E.F. The point of the touching is D, from which pointe I suppose the line D.B, to be dawen crosse the circle, and to deuide it into two cantles, where of the greater is B.A.D, and the lesser is B. C, D, and eche of them an angles drawen. for in the greater cantle the angle is by A. and is made of the right lines B,A. and A,D. in the lesser cantle the angle is by C. and is made of the right lines B,C. and C,D. Now saieth the Theoreme. that the angle B. D,F. is equall to the angle made in the cantle on the other side of the said line

line, that is to saye, in the cantle B.A.D, so that the angle B D.F, is equall to the angle B.AD., because the angle B.D.F is on the one side of the line B.,D (which is accordyng to the suppositiō of the Theoreme drawen crosse the circle) and the angle B.A.D, is in the cantle on the other side. Likewise the angle B.D.E, beyng on the one side of the line B.D, must be equall to the angle B.C.D, (that is the angle by C,) whiche is made in the cantle on the other side of the right line B D. The profe of al these I doe reserue, as I haue often said, to a conuenient boke, wherein they shall be all set at large.

¶The.lxxv. Theoreme.

In euery circle when two right lines doe crosse one an other, the likeiamme that is made of the portions of the one line, shall bee equall to the likeiamme made of the partes of the other line

¶Example.



Because this Theoreme doeth serue to many vses and would bée wel vnderstande, I haue set forthe two examples of it. In the first, the lines by their crossynge doe make their portiōs somwhat towarde an equalitie. In the seconde, the portions of the lines bée verye farre frō an equalitie, and yet in bothe these and in all other,

the Theoreme is true. In the first example the circle is A.B. C.D. in which the one line A.C, dooeth crosse the other line B.D, in the poincte E. now if you doe make one likeiamme, or longe square of D.E, and E.B, beyng the twoo portions of the line D.B, that longe square shall bee equall to the other long square made of A.E, and E.C, beyng the portions of the other line A.C. Likewise in the seconde example, the circle is F.G.H.K. in whiche the line F.H, dooeth crosse the other line G.K, in the poincte *L.* Wherefore if you make a likeiamme, or longe square of the twoo partes of the line F.H, that is to saie of F, L, and L.H, that long square will be equal to an other long square made of the twoo partes of the line G.K, which partes are G.L, and L.K. These longe squares haue I sette forthe vnder the circles, containyng their sides, that you maye somewhat whette your owne witte in practisyng this Theoreme, accordyng to the doctrine of the nineteneth conclusion.

¶ The.lxxvj. Theoreme.

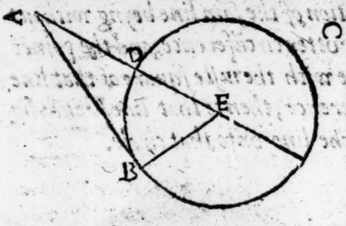
If a poincte be marked without a circle, and from that poincte twoo righte lines drawen to the circle, so that the one of them doe runne crosse the circle, and the other doe touche the circle onely, the long square that is made of that whole line which crosseth the circle, & the portion of it, that lieth betwene the vtter circumference of the circle and the poincte, shall bee equall to the full square of the other line, that onely toucheth the circle.

¶ Example.

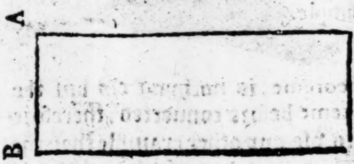
This circle is D.B.C, and the poinct without the circle is A, from whiche poincte there is drawen one line crosse the circle, and that is A.D.C, and an other line is drawen from

teh

the saied pricke, to the marge or edge of the circumference

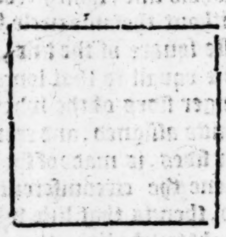


of the circle, and dooeth onely touch it, that is the line A. B. And of that first line A, D. C. you may perceiue one parte of it which is A. D. to lye with out the Circle, be, twene the vtter cicum: ference of it and the pointe assigned which was *A*.



Now concerning the mea ning of the Theoreme, if you make a long square of the whole line A.C. and of that parte of it that lieth betweene thecircumferēce and the pointe, which is A. D. that longe square shall be equall to the full square of the touch line A.B. according not onely at this figure sheweth, but also the saied nineteneth Conclusion doth proue, if you liste to examine the one by the other.

A B



The lxxvii. Theoreme.

If a pointe bee assigned without a circle, and from that pointe two right lines bee drawen to the circle, so that the one doe crosse the circle, and the other doe ende at the circumfe-

rence, and that the longe square of the line, whiche crosseth the circle made with the portion of the sam line beyng without the circle made, betwene the vtter circūference, and the poinct aßigned, dooe equally agree with the iuste square of that line that endeth at the circumference, then is that line so endyng on the circumference, a touche line vnto that circle.

¶Example.

In as muche as this Theoreme, is nothyng els but the sentence of the laste Theoreme before conuerted, therefore it shall not bée néedefull, to vse any other example then the same, for as in that other Theoreme, because the one line is a touche line, therefore it maketh a square iuste equall, with the longsquare made of that whole line, which crosseth the circle, and his portion liyng without the same circle. So saith this Theoreme: that if the iuste square of the line, that endeth on the circumference, bée equall to that longsquare, which is made as for his longer sides of the whole line, which commeth from the poincte assigned, and crosseth the circle, and for his other shorter sides, is made of the portion of the same line, liyng betwene the circumference of the circle, and the poincte assigned, then is that line which endeth on the circumference a right touche line, that is to say, if the full square of the right line A.B, bée equall to the long square, made of the whole line A. C. as one of his lines, and of his portion A.D, as his other line, then must it nedes bée, that the line A.B. is a right touch line vnto the circle D.B.C And thus for this tyme, I make an ende of the Theoremes.

FINIS.